Subject ______ موضوع الدرس

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Eulers equation :-

1) $\cos \underset{\text{Even}}{w_0} = \dfrac{e^{jw_0} + e^{-jw_0}}{2}$

2) $\sin \underset{\text{odd}}{w_0} = \dfrac{e^{jw_0} - e^{-jw}}{j2}$

3) $e^{jw_0} = \cos w_0 + j \sin w_0$ important

Time delay :-

$x(t) = \sin w_0 t$

$x(t - t_d) = \sin[w_0 (t - t_d)]$

$= \sin[w_0 t - w_0 t_d]$

$= \sin[w_0 t + \phi_0]$

$-w_0 t_d = \phi_0$

$\therefore \underset{\text{time delay}}{t_d} = -\dfrac{\overset{\text{phase delay}}{\phi_0}}{w_0}$

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Eulers equation :-

1) $\cos w_0 = \dfrac{e^{jw_0} + e^{-jw_0}}{2}$ Even

2) $\sin w_0 = \dfrac{e^{jw_0} - e^{-jw}}{j2}$ odd

3) $e^{jw_0} = \cos w_0 + j\sin w_0$ important

Time delay :-

$x(t) = \sin w_0 t$

$x(t - t_d) = \sin[w_0 (t - t_d)]$

$= \sin[w_0 t - w_0 t_d]$

$= \sin[w_0 t + \phi_0]$

$-w_0 t_d = \phi_0$

$\therefore t_d = -\dfrac{\phi_0}{w_0}$ phase delay

time delay

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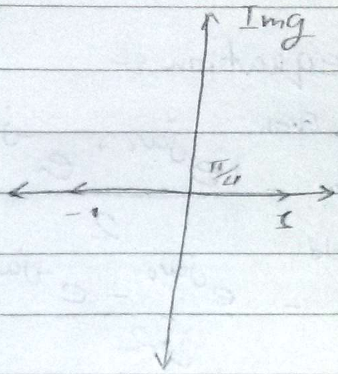
$e^{2j\pi} = +1$

$\boxed{1}$, $e^{+j\pi} = -1$

1 amplitude $e^{-j\pi} = -1$

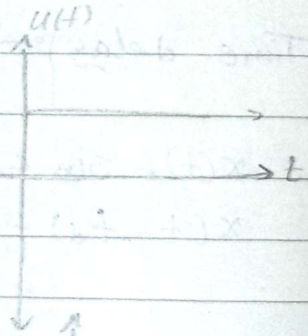
$e^{+j\pi/2} = +j$

$e^{-j\pi/2} = -j$

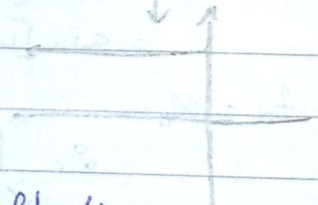


* unit step $u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & e.w \end{cases}$$

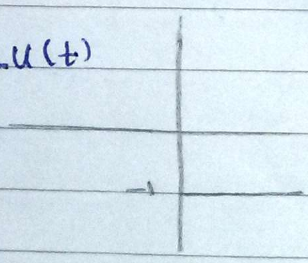


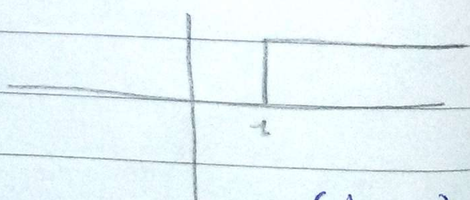
$$u(-t) = \begin{cases} 1 & t \leq 0 \\ 0 & e.w \end{cases}$$



إنعكاس reflection

$-u(t)$





$u(t-1)$

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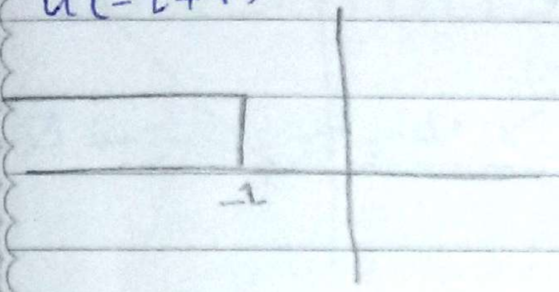
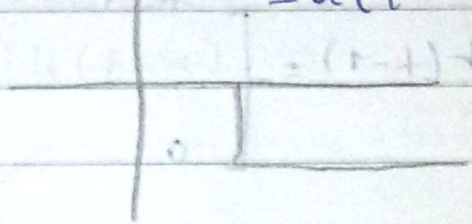
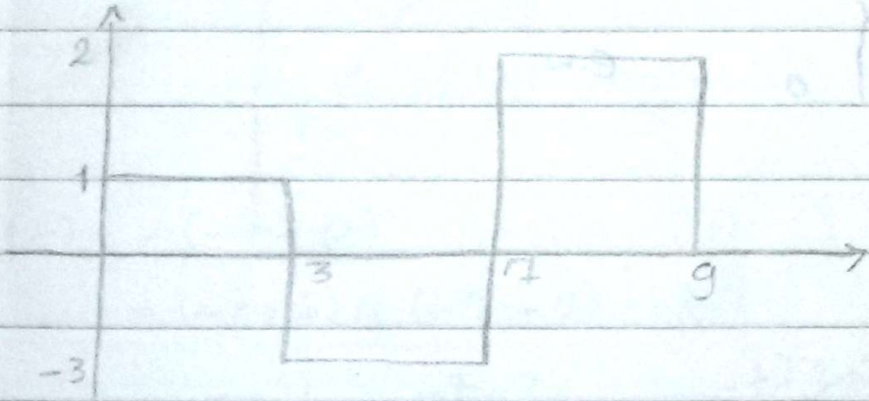
$u(-t+1)$



$-u(t-1)$



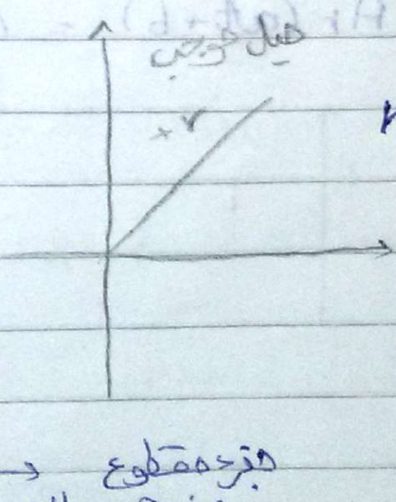


Left to Right =>

$$x(t) = u(t) - 4u(t-3) + 5u(t-7) - 2u(t-9)$$

* Ramp signal $r(t)$

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & e.w \end{cases}$$



$r(t)$

* note/ $f(x) = \underbrace{a}_{\text{slope}}x + b \longrightarrow$ جزء مقطوع من محور الصادات

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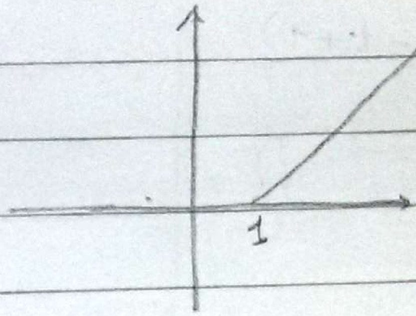
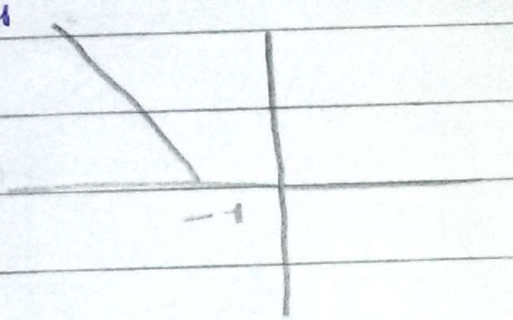
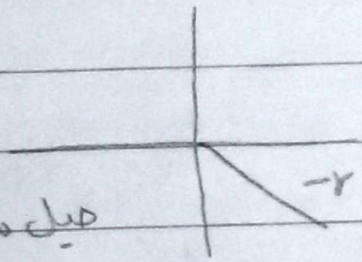
$$r(t-1) = \begin{cases} t-1 & t-1 \geq 0 \\ & t \geq 1 \\ 0 & e.w \end{cases}$$



$$r(t+1) = \begin{cases} -t+1 & t \leq -1 \\ 0 & e.w \end{cases}$$



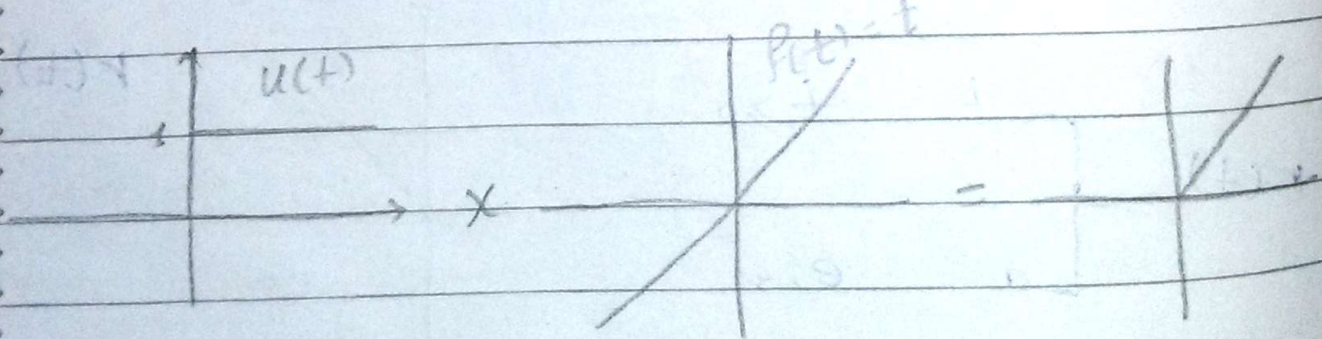


ميل سالب

① $r(t) = t\,u(t)$

② $A\,r(t) = A\,t\,u(t)$

③ $A\,r(at+b) = A(at+b)\,u(at+b)$



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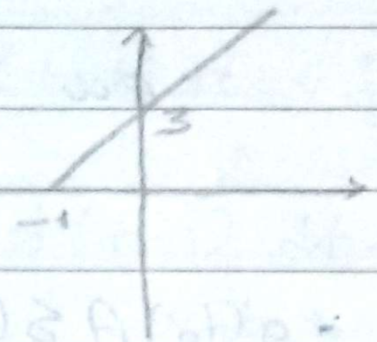
$$x(t) = 3r(t+1) = 3(t+1) \cdot u(t+1)$$
$$= (3t+3)\,u(t+1)$$

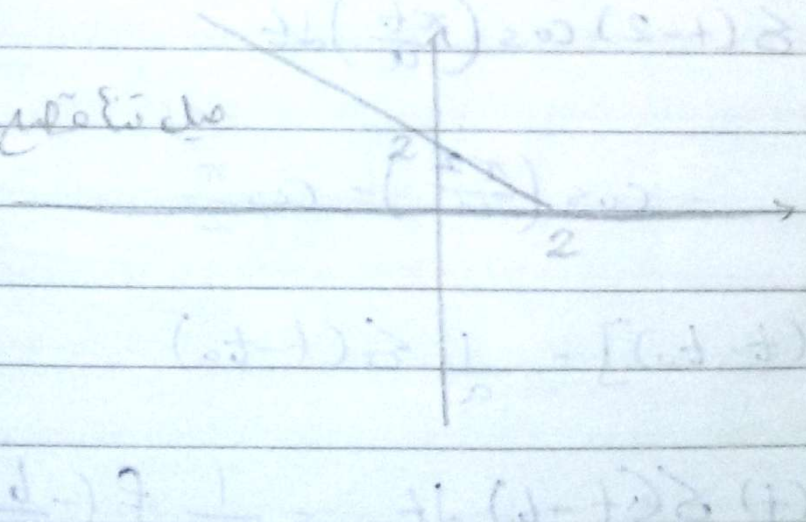
ميل موجب ويقطع عند 3



$$x(t) = r(-t+2)$$
$$= (-t+2)\,u(-t+2)$$

$$r(-t+2) = \begin{cases} -t+2 & -t+2 \geq 0 \\ & -t \geq -2 \\ & t \leq 2 \\ 0 & e.w \end{cases}$$

ميل تنازلي



Subject ______ موضوع الدرس

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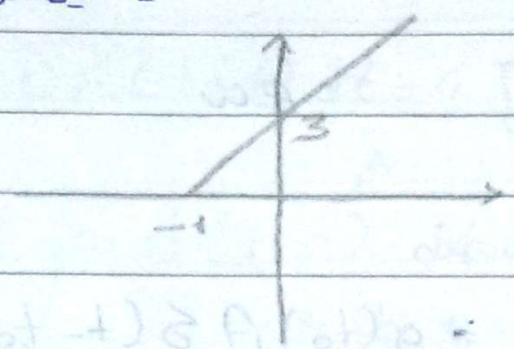
$$x(t) = 3r(t+1) = 3(t+1) \cdot u(t+1)$$

$$= (3t+3)\, u(t+1)$$

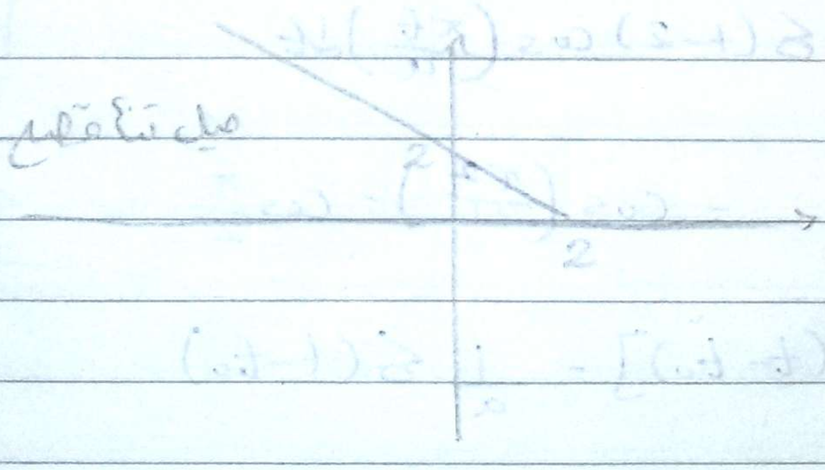
ميل موجب ويقطع عند 3



$$x(t) = r(-t+2)$$

$$= (-t+2)\, u(-t+2)$$

$$r(-t+2) = \begin{cases} -t+2 & -t+2 \geq 0 \\ & -t \geq -2 \\ & t \leq 2 \\ 0 & e.w \end{cases}$$



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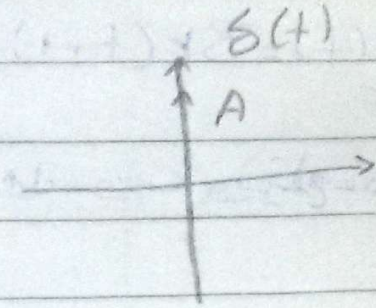
* Impulse function $\delta(t)$



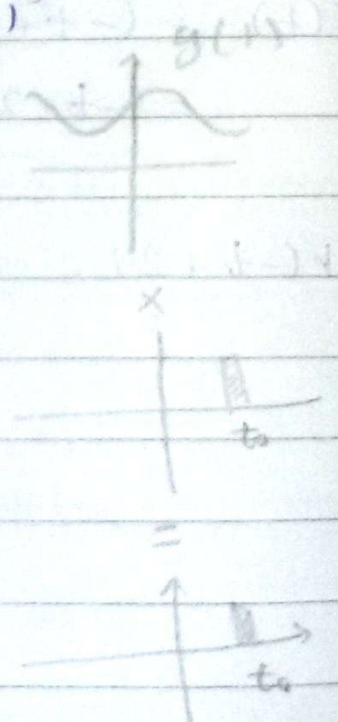
$$\delta(t) = \begin{cases} A & t=0 \\ 0 & e.w \end{cases}$$

① $g(t)\, A\, \delta(t-t_0) = g(t_0)\, A\, \delta(t-t_0)$

Ex :- ① $(t^3+3)\,\delta(t) = 3\,\delta(t)$

② $e^{-2t}\,\delta(t) = \delta(t)$

② $\int_{-\infty}^{\infty} g(t)\,\delta(t-t_0) = g(t_0)$



Ex :- $\int_{-\infty}^{\infty} \delta(t-2)\cos\left(\frac{\pi t}{4}\right)dt$

$= \cos\left(\frac{\pi 2}{4}\right) = \cos\frac{\pi}{2}$

③ $\delta[a(t-t_0)] = \frac{1}{a}\,\delta(t-t_0)$

④ $\int_{-\infty}^{\infty} f(t)\,\delta(at+b)\,dt = \frac{1}{|a|} f\left(-\frac{b}{a}\right)$

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Ex:- $\int_{-\infty}^{\infty} e^{-2(x-t)} \, \delta(2-t)\, dt = \frac{1}{|-1|} e^{-2(x-2)}$

Ex:- $\int_{0}^{7} (t+1)\, \delta(t+1)\, dt = 0$ [لأن -1 خارج حدود التكامل]

or => $\int_{0}^{7} (t+1)\, \delta(t-1)\, dt = 2$

$t = 1$

داخل التكامل المحدود

Subject: Lactcher 2# موضوع الدرس

Date: / / الموافق / / التاريخ

Amplitude scale $\longrightarrow$ Time Scale $\longrightarrow$ Time shift $\rightarrow$ reversal

Time.

① ~~Amplitude~~ Scale :-

$$y(t) = x(at)$$

$|a| > 1 \longrightarrow$ compress

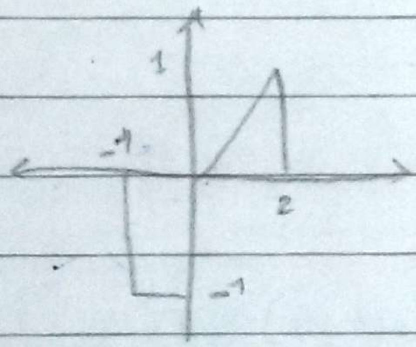
$|a| < 1 \longrightarrow$ Expand

$\frac{dr(t)}{dt} \rightarrow u(t)$

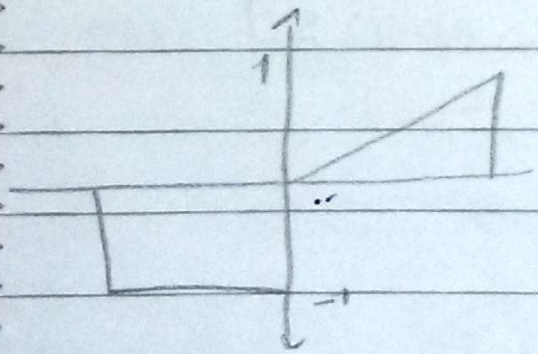
$\frac{du(t)}{dt} \rightarrow \delta(t) = 1$

$\int \delta(t) = 1 = u(t)$

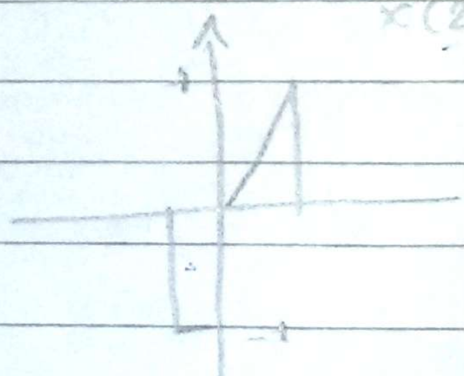
$\int u(t) = r(t)$



x(0.1t)



x(2t)



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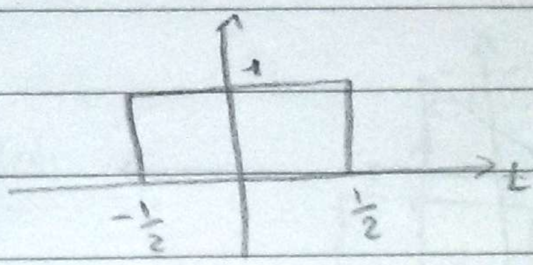
Ex) if $g(t) =$

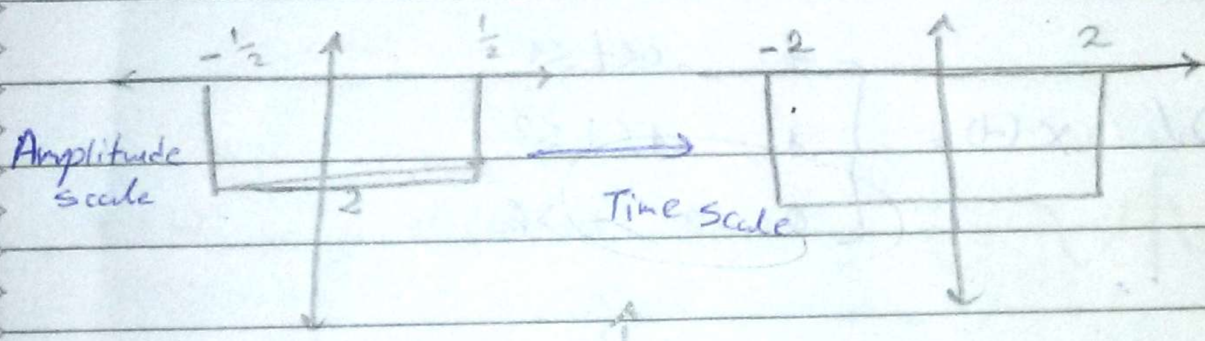


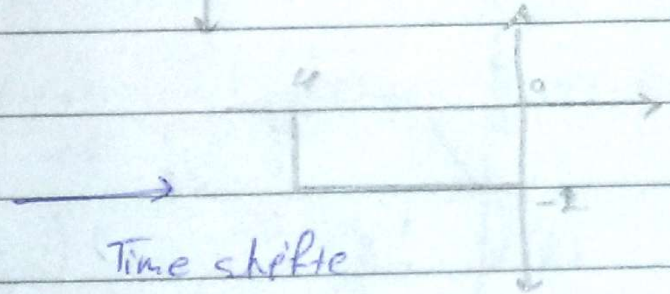
find $-2g\left(\frac{t+2}{4}\right)$?

$A\left(\frac{t-t_0}{a}\right)$

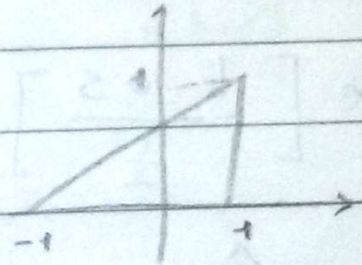
$A = -2$

$t_0 = -2 \quad a = 4$





Ex②/ find $3g(-2t-1)$



نغيرها زي المفهوم

ازاحة الانعكاس

$3g(-2[t+0.5]) \Rightarrow 3g\left[\frac{-t+0.5}{\frac{1}{2}}\right]$

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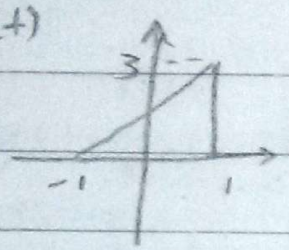
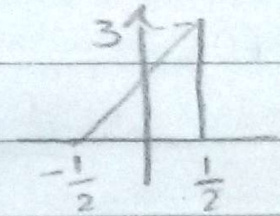
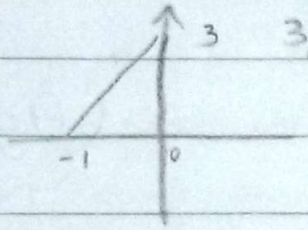
$3g(t)$

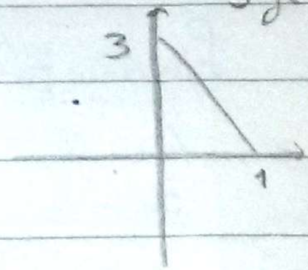




$3g(2t)$

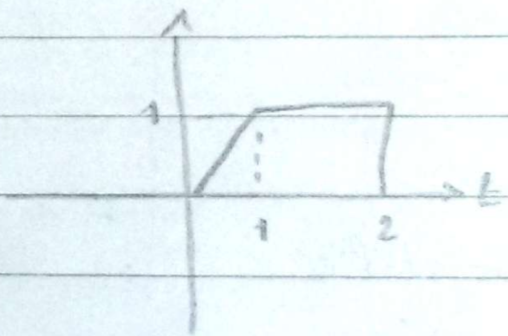


$3g\left(\frac{t+0.5}{\frac{1}{2}}\right)$



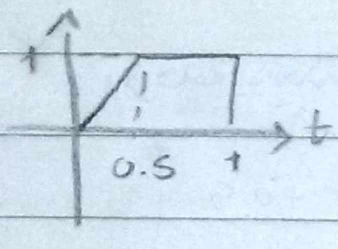
$3g\left(-\frac{t+0.5}{\frac{1}{2}}\right)$

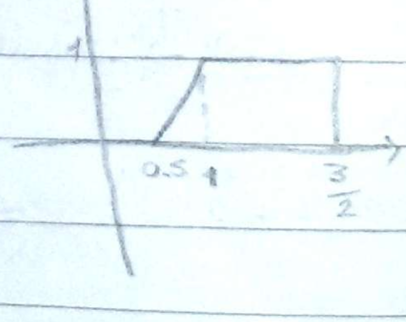
Ex ③ / $x(t) = \begin{cases} t & 0 \le t \le 1 \\ 1 & 1 \le t \le 2 \\ 0 & t > 2 \end{cases}$



$x[2(t-0.5)]$

$x\left[\frac{t-0.5}{\frac{1}{2}}\right]$





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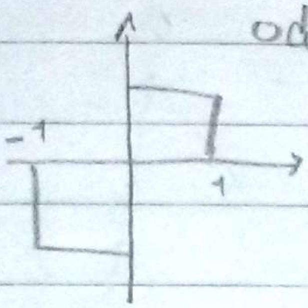
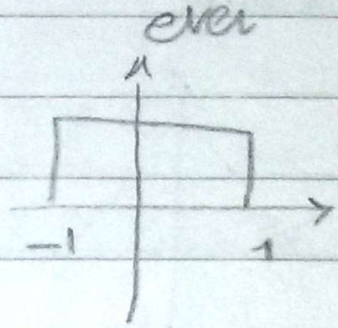
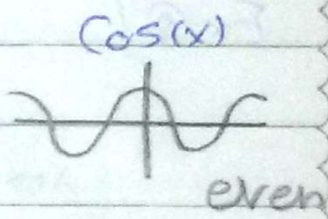
odd / Even



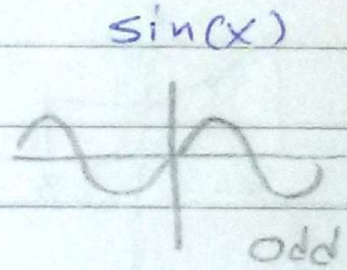


$x(t) = -x(-t)$

$x(t) = x(-t)$

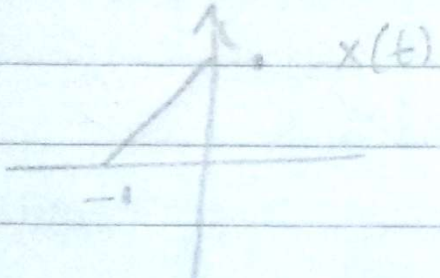


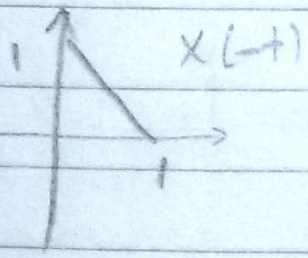
$$x_{even}(t) = \frac{1}{2} x(t) + \frac{1}{2} x(-t)$$

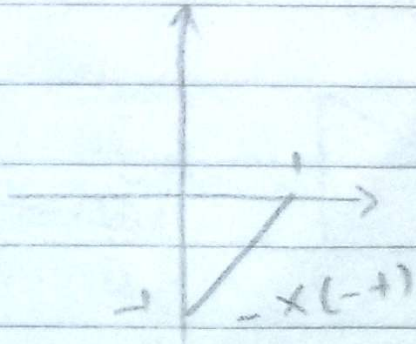


$$x_{odd}(t) = \frac{1}{2} x(t) - \frac{1}{2} x(-t)$$

Ex)







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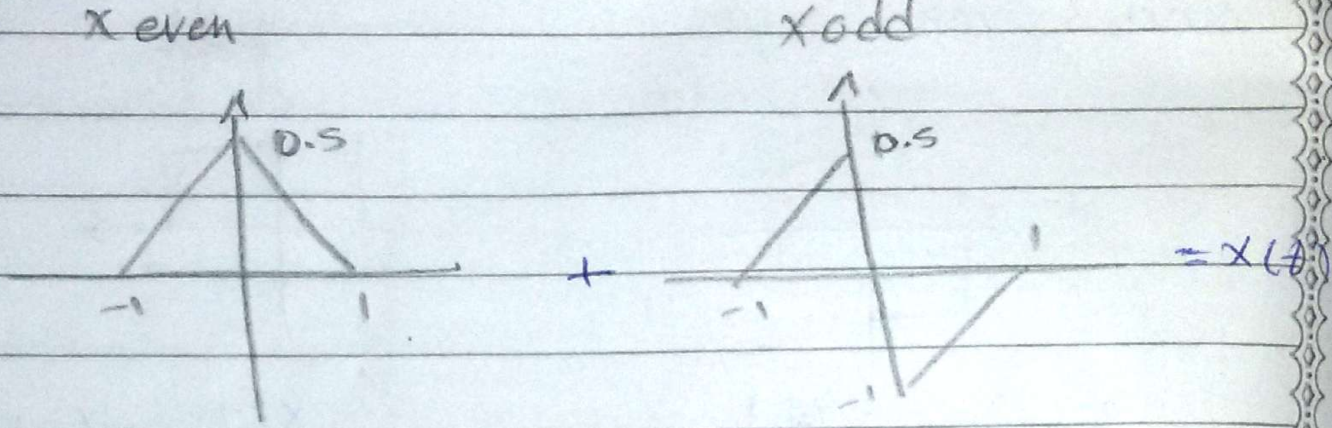
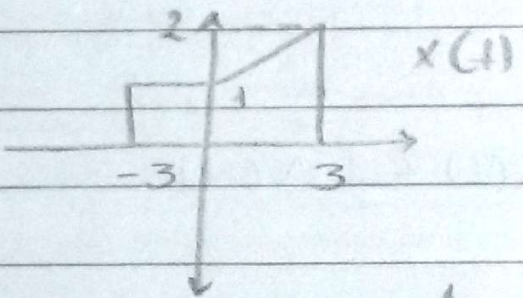
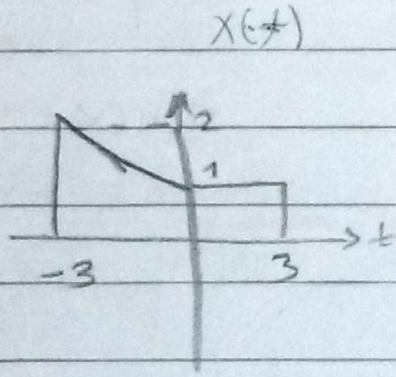
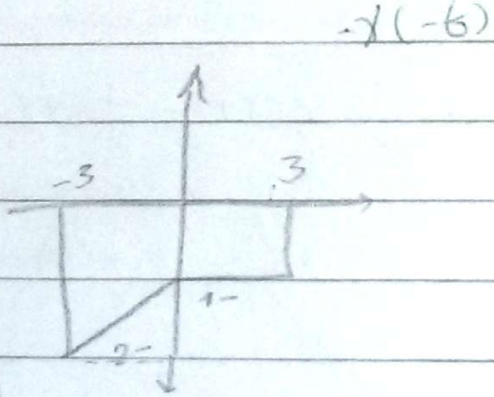
x even + x odd = x(t)

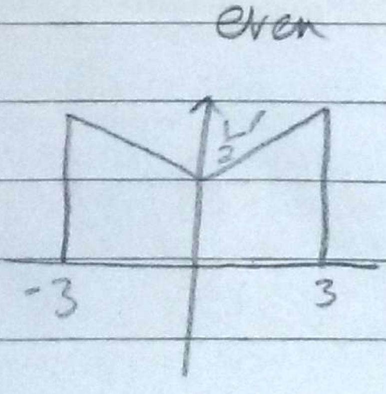


Ex ②:









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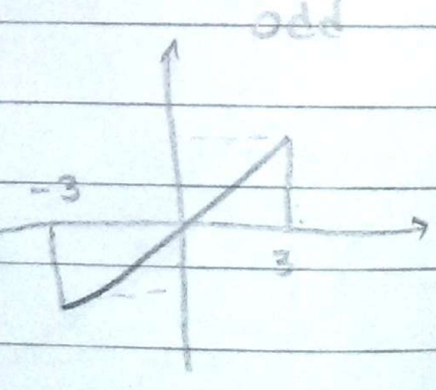

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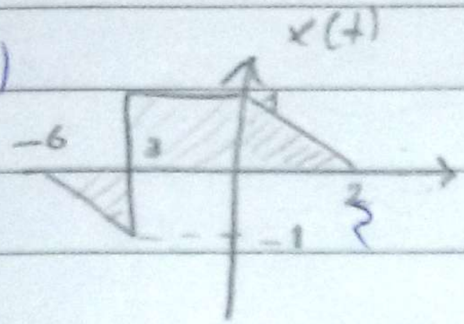
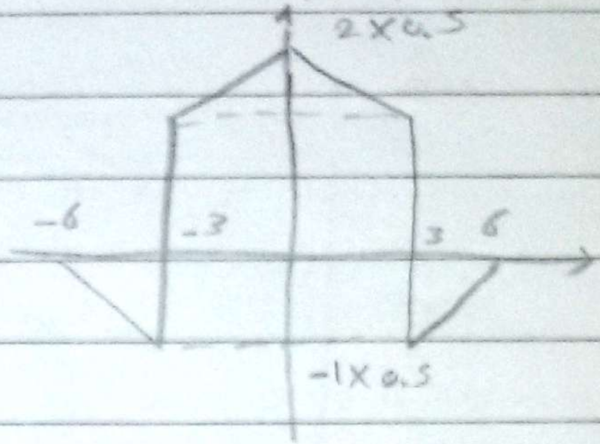
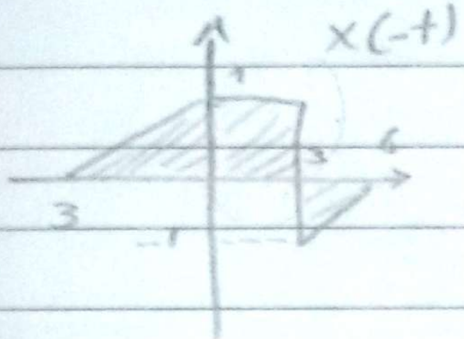
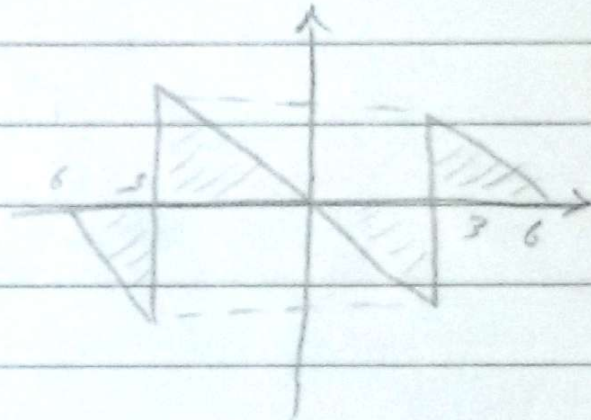
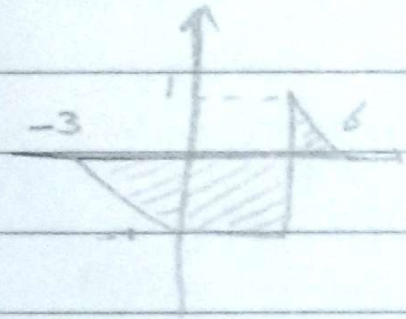
Ex)











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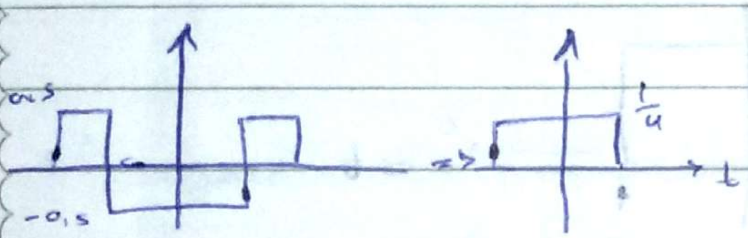
Power and energy signal /

energy signal   power signal

$E = \int_{-\infty}^{\infty} x^2(t)\, dt =$ $P =$



لو طلعت معدلة تخلها في تكامل

لو طلعت قيمة يعني Energy

لو 0 أو ∞ يعني حاجة تانية

Ex/ $x(t) = e^{-\alpha t} u(t)$ [تربيع ما يأثرش في $u(t)$]

$$\int_{-\infty}^{\infty} e^{-2\alpha t} u(t)\, dt = \int_{0}^{\infty} e^{-2\alpha t}\, dt = \frac{1}{-2\alpha} \int_{0}^{\infty} -2\alpha e^{-2\alpha t}\, dt$$

$$= \frac{1}{-2\alpha} \left(e^{-2\alpha t} \Big|_{0}^{\infty} \right) = \frac{1}{-2\alpha} (0 - 1)$$

$$= \frac{1}{2\alpha}$$

energy

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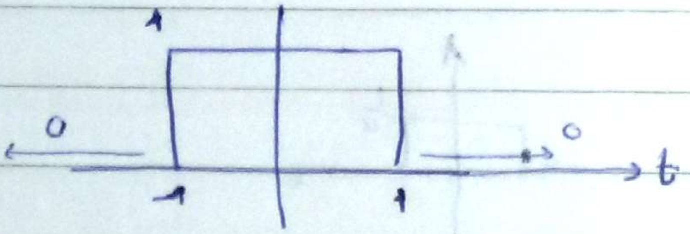
$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)\,dt)$$

$P = \square \neq 0 \neq \infty$ in Periodic signal $\cancel{\lim_{T\to\infty}}$

$$\frac{1}{T}\int_0^T x^2(t)\,dt$$



$$\because P = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-1}^{1} (1)^2\,dt = \lim_{T\to\infty} \frac{1}{T}[2]$$

$$= \lim_{T\to\infty} \frac{2}{T} = 0$$

~~Cos~~ $x(t) = A\cos(\omega t)$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^T A^2\cos^2\omega t\,dt$$

$$= \lim_{T\to\infty} \frac{A^2}{T}\int_0^T \frac{1}{2}[1+\cos 2\omega t]\,dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T}\left[\int_0^T dt + \int_0^T \cos(2\omega t)\,dt\right]$$

0

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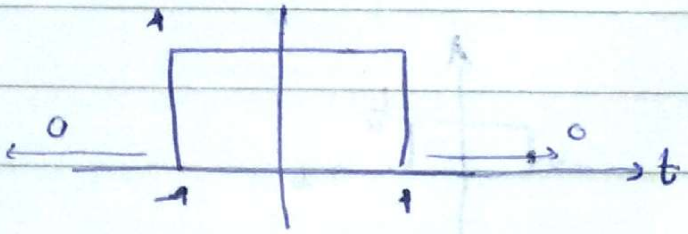
$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)\,dt$$

$P = \square \neq 0 \neq \infty$ in periodic signal $\cancel{\lim_{T\to\infty}}$ $\frac{1}{T}\int_0^T x^2(t)\,dt$



$$\therefore P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-1}^{1} (1)^2\,dt = \lim_{T\to\infty} \frac{1}{T}[2]$$

$$= \lim_{T\to\infty} \frac{2}{T} = 0$$

~~cos~~ $x(t) = A\cos(\omega t)$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_0^T A^2 \cos^2 \omega t\,dt$$

$$= \lim_{T\to\infty} \frac{A^2}{T} \int_0^T \frac{1}{2}[1 + \cos 2\omega t]\,dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \left[\int_0^T dt + \underbrace{\int_0^T \cos(2\omega t)\,dt}_{0}\right]$$

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$$= \lim_{T\to\infty} \frac{A^2}{2T}\int_0^T dt = \lim_{T\to\infty} \frac{A^2}{2\cancel{T}} \cancel{T} = \frac{A^2}{2}$$

$\therefore$ P $\sin(\omega t) = \frac{A^2}{2}$ للكل

$x(t) = A\sin\omega_0 t + B\cos\omega_0 t$

$P = \frac{A^2}{2} + \frac{B^2}{2}$.

* if signal ~~E=~~ $E = 0$ or ∞

and $P = 0$ or ∞

يعني signal مش الزوز

* $x(t) = A$

$$E = \int_{-\infty}^{\infty} A^2\,dt = \infty.$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^T A^2\,dt \Rightarrow A^2$$

* $x(t) = A + B\cos(\omega t)$

$$P = A^2 + \frac{B^2}{2}$$

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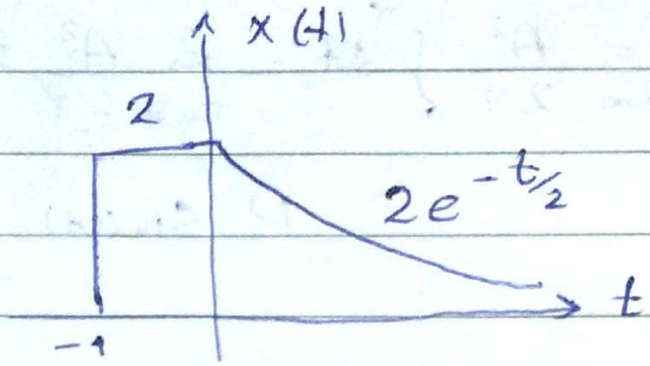
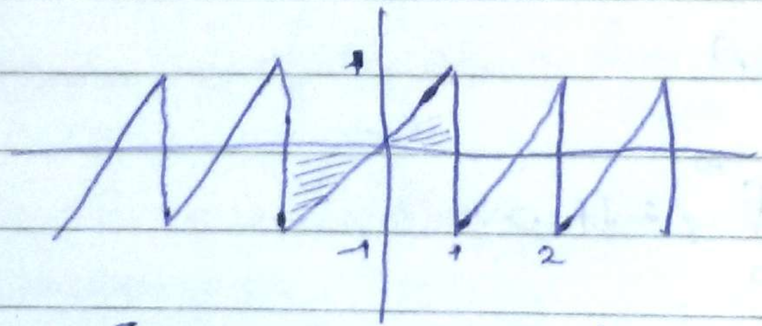


$$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-1}^{0} 4\,dt + \int_{0}^{\infty} 4e^{-t}\,dt$$

$$= 4[t]_{-1}^{0} + -4[e^{-t}]_{0}^{\infty}$$

$$= 4[0-(-1)] + (-4)[0-1] = 8$$



$$P = \frac{1}{T}\int_0^T x^2(t)\,dt = \frac{1}{\cancel{T}_{2}}\int_{-1}^{1} t^2\,dt$$

$$= \frac{1}{2}\left[\int_{-1}^{0} t^2\,dt + \int_{0}^{1} t^2\,dt\right] = \frac{1}{3}$$

Subject Anas Allosh # موضوع الدرس
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Test ②

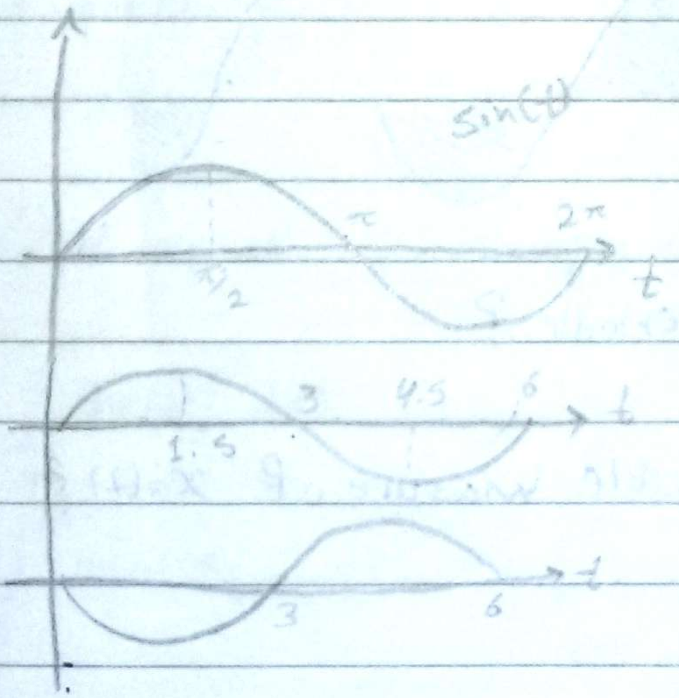
$\sin(\omega_0 t - \phi_0)$
$t_d = -\frac{\phi_0}{\omega_0}$

Q1 $x_1(t) = \sin(\frac{\pi}{3} t - \pi)$

$x_2(t) = u(t-3) - u(t-9) + u(t-10.5) - u(t-13.5)$

① Find $x_1(t)$ delay time?

② Find and sketch $x_3(t) = x_2(t)\, x_1(t)$



Sin(t)

$\sin(\frac{\pi}{3} t)$

$\sin(\frac{\pi}{3} t - \pi)$

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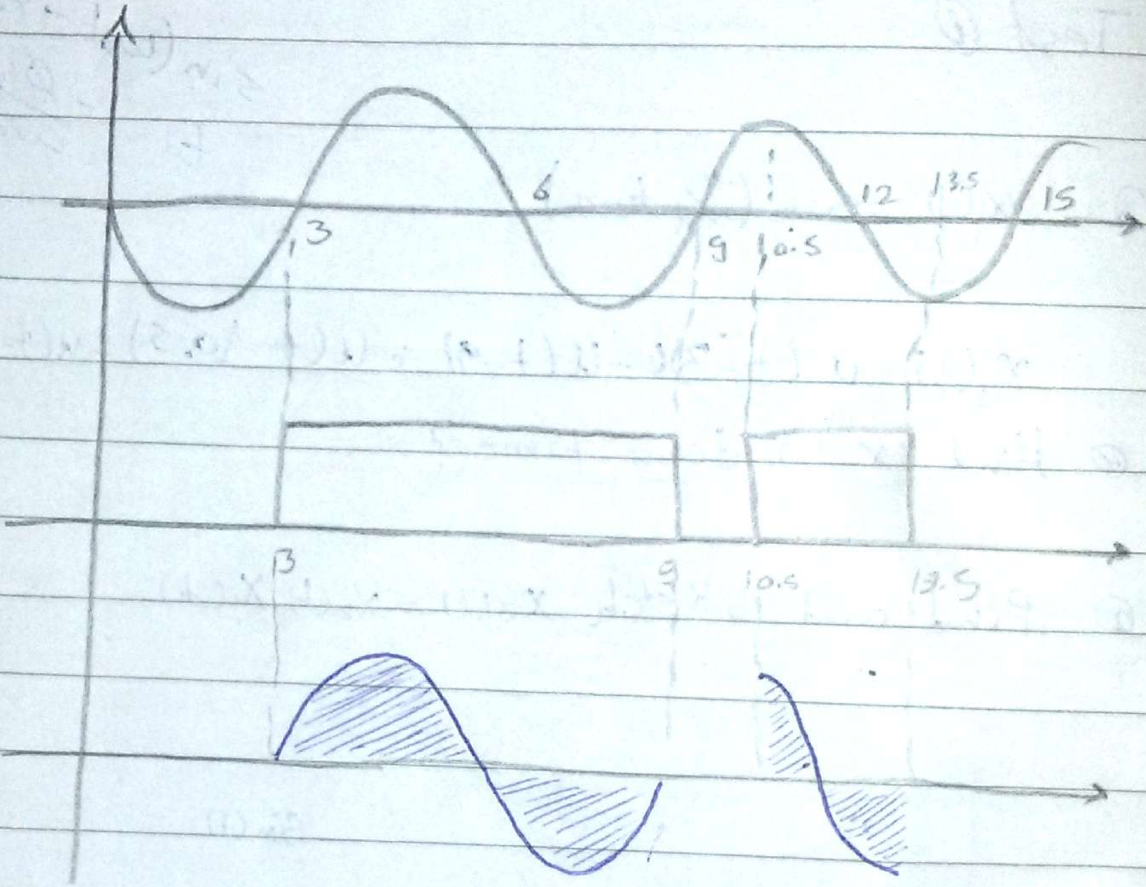

③ Is $x_3(t)$ periodic ?

④ find a suitable miasure of $x_3(t)$?

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① $\omega_0 = \pi/3$ $\quad \phi_0 = -\pi$

$$t_d = \frac{-(-\pi)}{\pi/3} = 3$$

② $\omega = \pi/3$, $\omega = \frac{2\pi}{T}$ $\quad T = 6$

③ Not periodic

④ limited signal => energy signal

$$E = \int |x(t)|^2 dt$$

$$E = \int_3^9 |\sin(\tfrac{\pi}{3}t - \pi)|^2 dt + \int_{10.5}^{13.5} |\sin(\tfrac{\pi}{3}t - \pi)|^2 dt$$

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Q2 (a) Find odd/Even parts of given $x(t)$

$$\text{Even part} = \frac{1}{2}[x(t) + x(-t)]$$

$$\text{odd part} = \frac{1}{2}[x(t) - x(-t)]$$

$\boxed{b}$ Find:-

$$\int_{-\infty}^{\infty} [u(t-6) - u(t-10)] \sin\left(\frac{3\pi t}{4}\right) \delta(t-5) dt$$

$$\int_{6}^{10} \sin\left(\frac{3\pi t}{4}\right) \delta(t-5) dt = 0$$

Notes/ الأجزاء المكملة / الميول تجمع

+1/

+3/

+4/

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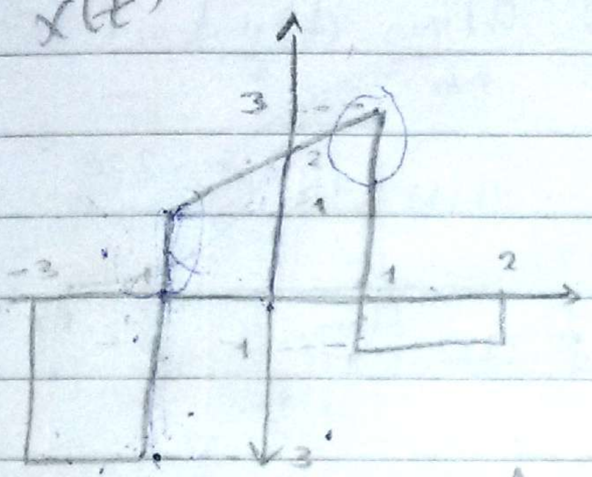
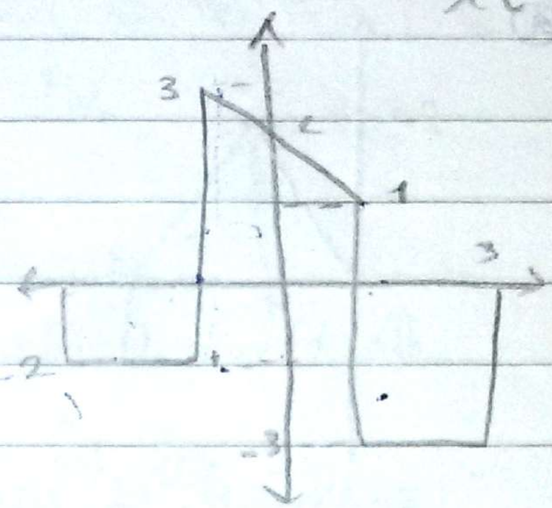
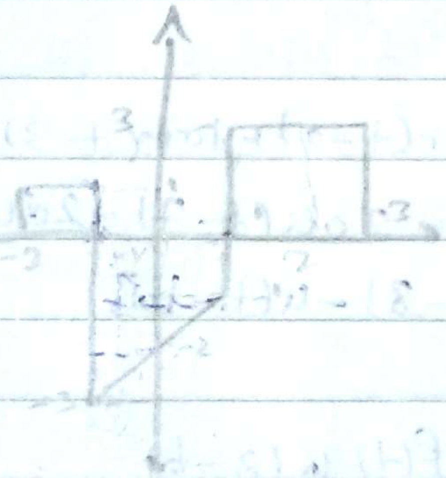
x(t)

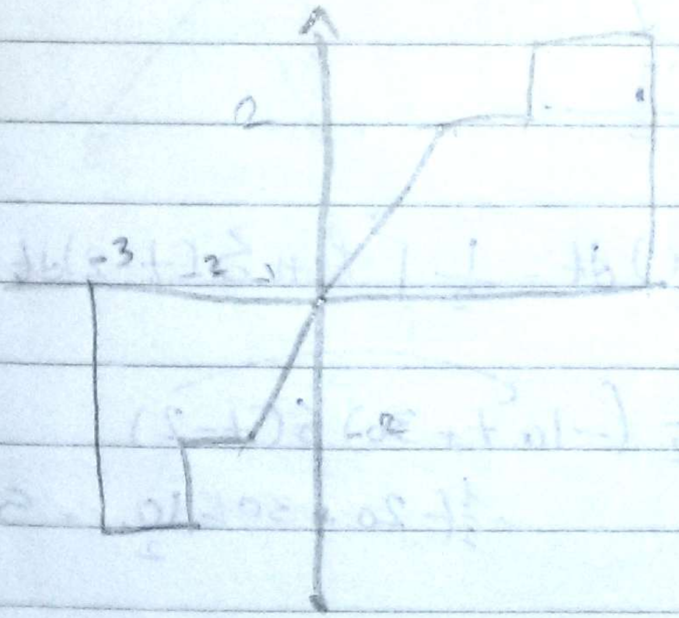


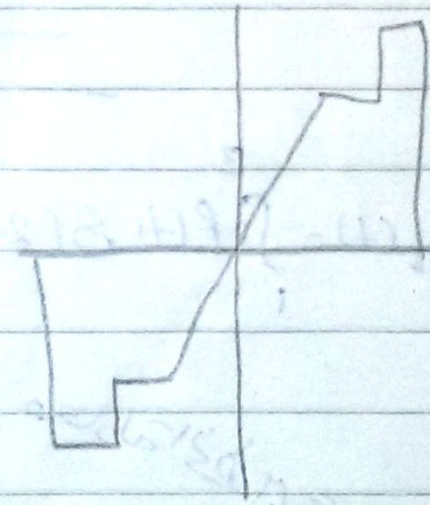
x(-t)





-x(-t)





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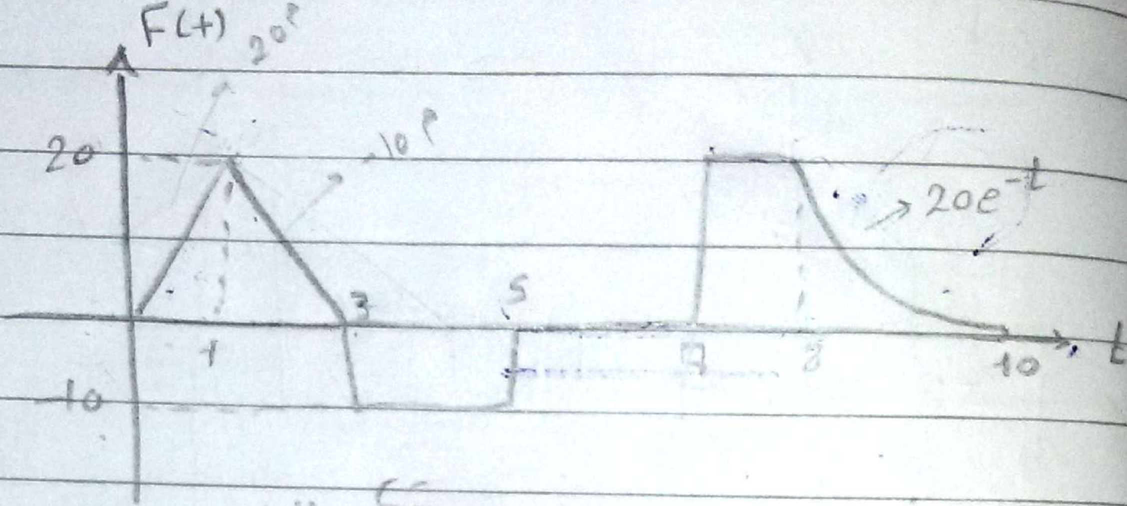
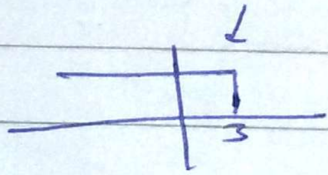
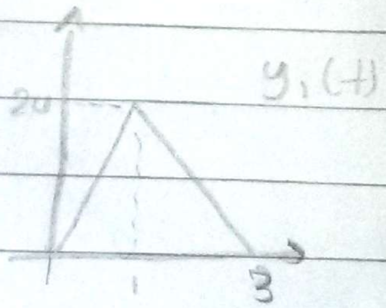
Q (a)



$f(t) = 20r(t) - 30r(t-1) + 10r(t-3) - 10u(t-3)$
$+ 10u(t-5) + 20u(t-7) - 20u(t-8) +$
$20e^{-t}[u(t-8) - u(t-10)]$

(b) find $y_1(t) = f(t)\,u(3-t)$ =





$y_2(t) = \int_1^3 f(t)\,\delta(2t-4)\,dt = \frac{1}{2}\int_1^3 f(t)\,\delta(t-2)\,dt$

معادلة الخط المستقيم $\frac{1}{2}(-10t + 30)\,\delta(t-2)$

$= \frac{1}{2}(-20 + 30) = \frac{10}{2} = 5$

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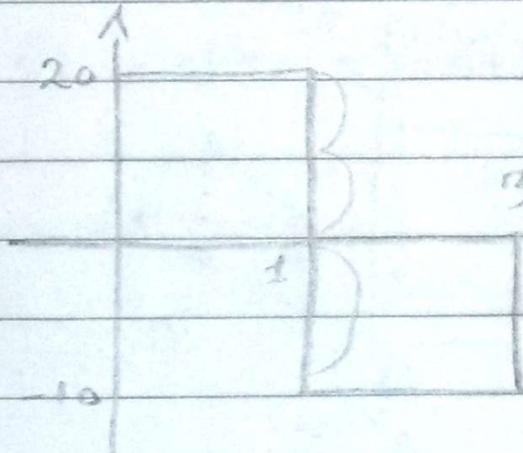
③ Find $y_3(t) = \frac{dy_1(t)}{dt}$

$\because \frac{\partial r(t)}{\partial t} = u(t)$

$\therefore \frac{\partial}{\partial t}\left[20r(t) - 30r(t-1) + 10r(t-3)\right]$

$y_3 = 20u(t) - 30u(t-1) + 10u(t-3)$



$= 20u(t) - 20u(t-1)$

$-10u(t-1) + 10u(t-3)$

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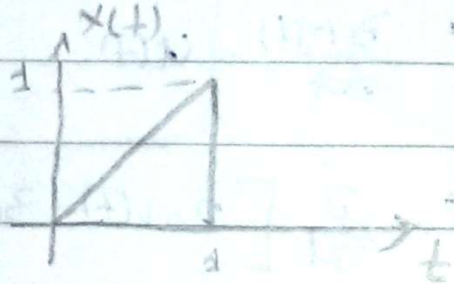
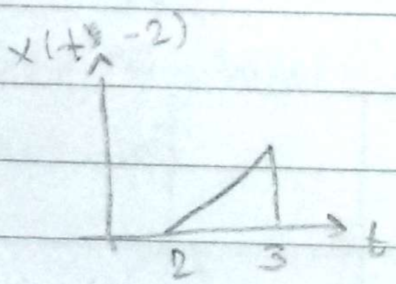
find E of ① $x(t-2)$

② $x(3t)$

③ $2x(t)$



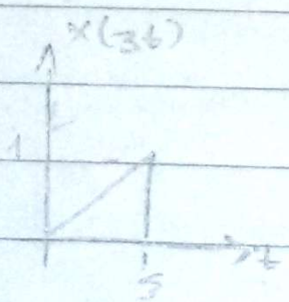
$$E = \int_0^1 |t|^2\,dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}$$



ماعملت ال shift

في ال E

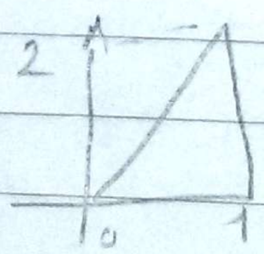
$E = \frac{1}{3}$



$$E = \int_0^{\frac{1}{3}} |3t|^2\,dt$$

$3 \div \frac{1}{3}$

$$= \frac{9t^3}{3}\Big|_0^{\frac{1}{3}} = \frac{1}{9}$$



$$E = \int_0^1 |2t|^2\,dt$$

$$= \frac{4t^3}{3}\Big|_0^1 = 4\left(\frac{1}{3}\right)$$

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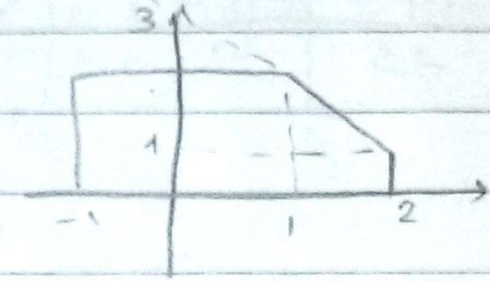
Plot signal /

① $x(t) = \begin{cases} 2 & |t|<1 \\ 3-t & 1<t<2 \\ 0 & e.w \end{cases}$



② $x(t) = r(3-0.5t)$

$= (3-0.5t)u(3-0.5t)$

$\longrightarrow \begin{cases} -0.5t+3 \\ 0 & o.w \end{cases}$

$-0.5t+3>0$
$-0.5t>-3$
$0.5t<3$
$t<6$

Find :-

3 خارج التكامل

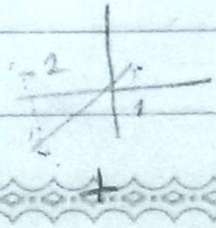
① $\int_{-2}^{1} (t+t^2)\,\delta(t-3)\,dt = 0$

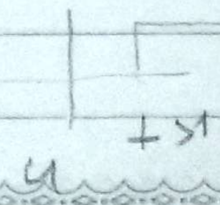
② $\int_{0}^{3} e^{t-2}\,\delta(2t-2) =$

$\frac{1}{2}\int_{0}^{3} e^{t-2}\,\delta(t-2) = \frac{1}{2}$

$u(t-1)$

③ $\int_{-2}^{1} t\,u(2t-2)\,dt = 0$





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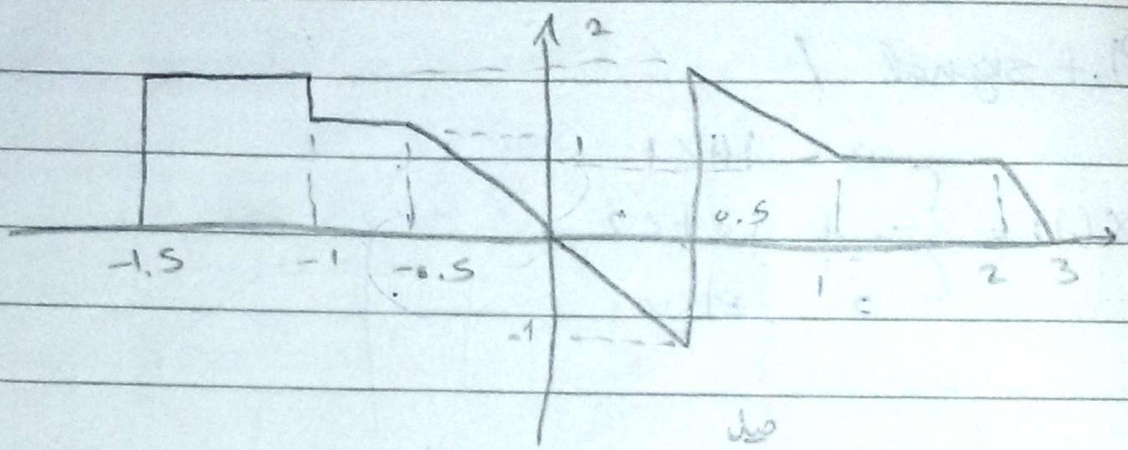

$x(t) = 2u(t+1.5) - u(t+1) - 2r(t+0.5) + 2r(t-0.5)$
$+ 3u(t-0.5) - 2r(t-0.5) + 2r(t-1) - r(t-2)$
$+ r(t-3)$

~~Q = x(t) = r(t) + 3r(t+3) + u(t) - 3r(t+1) - u(t-2)~~
~~= 3r(t+3) - 3r(t+1) + u(t) - u(t-2)~~

Form 1

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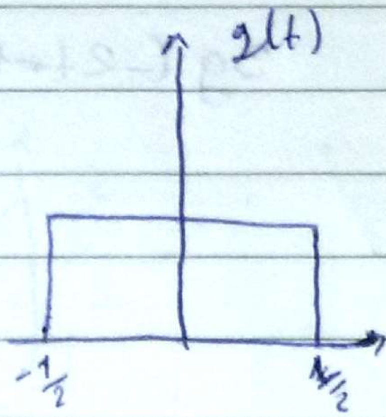
Ex) Given $g(t)$



find $-2g\left(\frac{t-2}{4}\right)$

① Amp scale

② Time scale

③ Time shift

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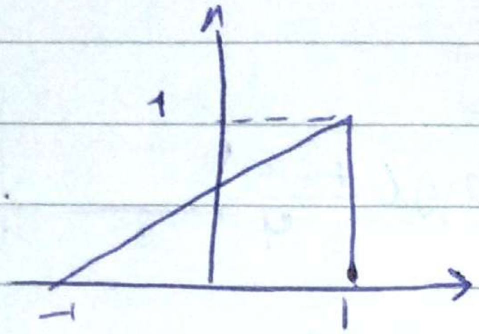
find $3g(-2t-1)$

Given



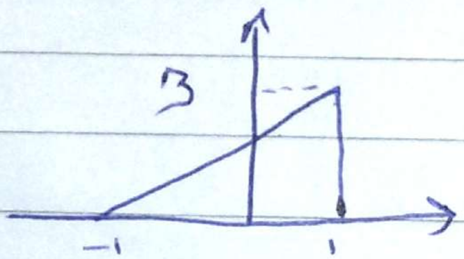
$3g(-2t-1)$ نقارنها ب $Ag\left(\frac{t-t_0}{a}\right)$

$\rightarrow 3g\left(-2\left(t+\frac{1}{2}\right)\right)$

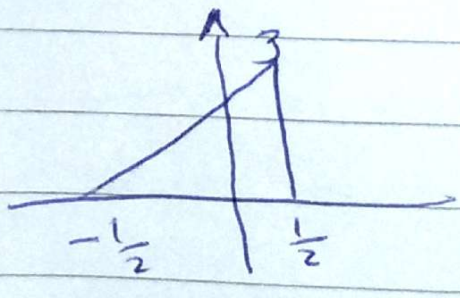
$3g\left(-\frac{\left(t+\frac{1}{2}\right)}{\frac{1}{2}}\right)$

a

① Amlitude scale:-



② Time scale



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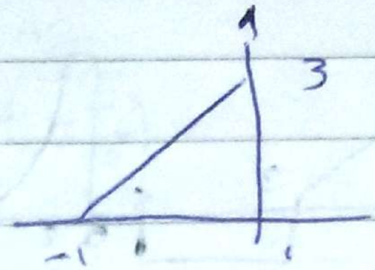
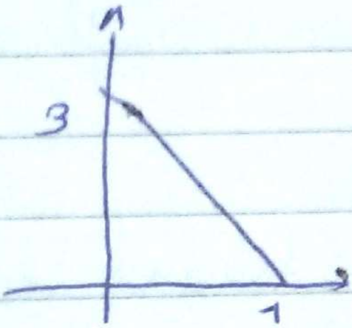
③ Time shift



④ Reverse



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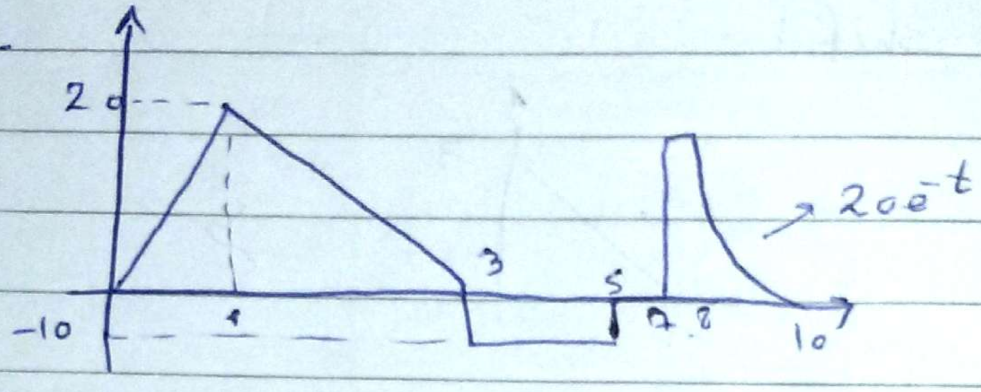
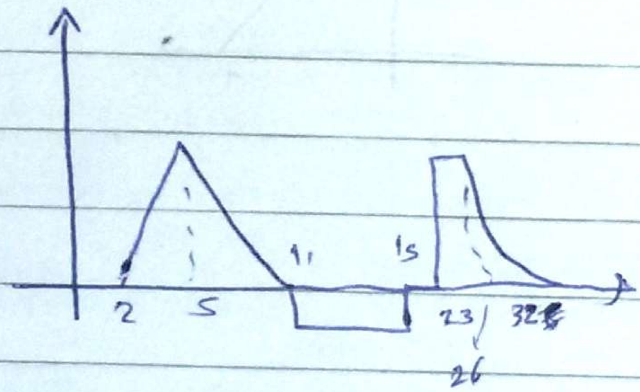
Q.

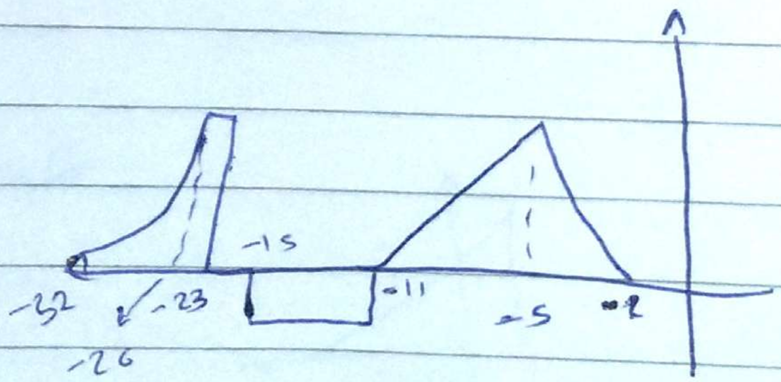


Find$\left(\frac{2-t}{3}\right)$

$f\left(-\frac{[t-2]}{3}\right)$



Time reverse (−)



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Q find a suitable measure for

$$\overset{\uparrow DC}{5} + \overset{\uparrow AC}{10\cos\left(\omega t + \frac{\pi}{3}\right)}$$

① $5 + 10\cos\left(\omega t + \frac{\pi}{3}\right)$

DC ⇐ الباور تربع

$\sin \& \cos = \frac{A_m^2}{2}$

$$P = 25 + \frac{10^2}{2}$$

② $10e^{j\omega t} \longrightarrow \cos\omega t + j\sin\omega t$

∵ الباور مش تخيلي

∴ $\cancel{j\sin\omega t} = 0$

$$\text{so} \quad \frac{10^2}{2} = 50 = P$$

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$a\cos(\omega_0 t) + b\sin(\omega_0 t) = C\cos(\omega_0 t + \theta)$

$$C = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Q - $x_1(t) = \sin(2\pi t) + 2)$

$x_2(t) = 1 - 2\sin(2\pi t)$

find @ $x_3(t) = x_1(t) \cdot (x_2)(t)$

$x_3(t) = \sin(2\pi t - 2\sin^2(2\pi t) + 2 - 4\sin(2\pi t)$

$x_3(t) = -3\sin(2\pi t) - 2\sin^2(2\pi t) + 2$

$\because 2\sin^2\theta = 1 - \cos 2\theta$

$= -3\sin(2\pi t) + 1 + \cos(4\pi t)$

$\omega_1 = \frac{2\pi}{T_1}$ $\omega_2 = \frac{2\pi}{T_2}$

$T_1 = 1$ $T_2 = \frac{1}{2}$

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$$\frac{T_1}{T_2} \quad \frac{1}{1/2} = 2 \qquad \text{periodic}$$

$1 \rightarrow$ rational

~~t~~ Irrational

$$\therefore P = (1)^2_{DC} + \left(\frac{3}{2}\right)^2 + \frac{1}{2}^2 = 6$$

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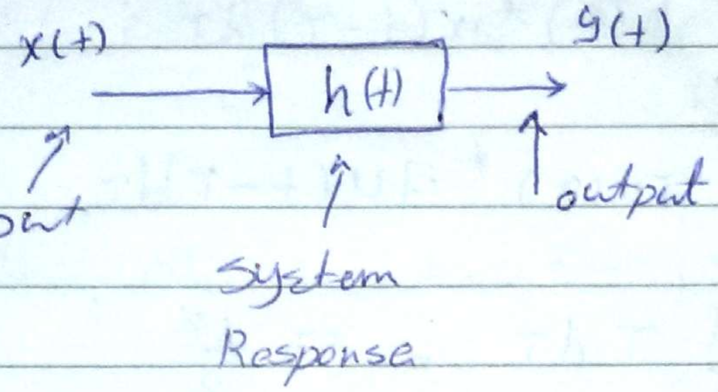
* Impulse Response [convolution]



$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

$$\hookrightarrow \int_{-\infty}^{\infty} h(t-\tau)\,x(\tau)\,d\tau$$

Choose the simpler function among $x(t)$, $h(t)$ and replace evey $t \rightsquigarrow (t-\tau)$

Ex) Given $h(t) = \text{ramp}(t)$ $\quad x(t) = 7u(t)$

find $y(t)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

نختار ال shift للدالة الأسهل فمثلاً هنا $x(t) = 7u(t)$

$$\therefore \; y(t) = \int_{-\infty}^{\infty} h(t) * x(t)$$

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$$= \int_{-\infty}^{\infty} h(\tau) * x(\tau)$$

$$= \int_{-\infty}^{\infty} h(\cancel{t}\,\tau) * x(t-\tau)\, d\tau$$

$$= \int_{0\;-\infty}^{t\;\infty} \tau u(\tau) * 7u(t-\tau) d\tau$$

$$7 \int_{0}^{t} \tau \, d\tau = 7 \frac{t^2}{2}$$

$$= 7 \frac{t^2}{2} u(t)$$

* ضربنا النتيجة في u(t) لانها توضح الفترة الزمنية اللي منها المخرج

وانه نفسها لحدود التكامل.

Ex2) Given $x(t) = e^{-t} u(t)$, $h(t) = e^{-2t} u(t)$

find $y(t)$?

$$\int_{-\infty}^{\infty} \underbrace{e^{-2\tau} u(\tau)}_{h(t)} \, e^{-(t-\tau)} \underbrace{u(t-\tau)\, d\tau}_{x(t)}$$

$$\int_{0}^{t} e^{-2\tau} e^{-t} e^{\tau} d\tau$$

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$$= e^{-t}\int_0^t e^{-\tau}\, d\tau$$

$$e^{-t}\left(-e^{-\tau}\Big|_0^t\right) = e^{-t}\left[-(e^{-t}-1)\right] u(t)$$

#

* system Classification :-

① linear / non linear / superposition

① when input $= x_1(t) \longrightarrow$ output is $y_1(t) = x_1(t)$

② when " $= x_2(t) \longrightarrow$ output is $y_2(t) = x_2(t)$

③ when " $= x_3(t) \longrightarrow$ " " $y_3(t) = x_3(t)$

and $x_3(t) = x_1(t) + x_2(t)$

if ① + ② = ③ then the system is linear.

if ① + ② ≠ ③ " " " is nonlinear.

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Ex) ① $y(t) = x^2(t)$? is this linear system?

$$x_1(t) \longrightarrow y_1(t) = x_1^2(t)$$

$$x_2(t) \longrightarrow y_2(t) = x_2^2(t)$$

$$x_3(t) \longrightarrow y_3(t) = x_3^2(t)$$

$$x_1^2(t) + x_2^2(t) \neq [x_1(t) + x_2(t)]^2 \quad \text{nonlinear}$$

② $y(t) = \ln(x)$?

$$\ln(x_1) + \ln(x_2) \neq \ln(x_1 + x_2) \qquad \text{non linear}$$

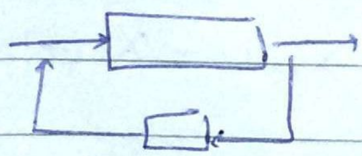
$\ln(x_1) + \ln(x_2)$ تساوي فقط $\ln x_1 \cdot \ln x_2$

③ $\dfrac{dy}{dt} = a y(t) + x(t)$?

$$x_1(t) \longrightarrow y_1'(t) = a y_1(t) + x_1(t)$$

$$x_2(t) \longrightarrow y_2'(t) = a y_2(t) + x_2(t)$$

$$x_3(t) \longrightarrow y_3'(t) = a y_3(t) + x_3(t)$$

feedback ⇒ 

$$a y_1(t) + x_1(t) + a y_2(t) + x_2(t) = a y_3 + x_3(t)$$

$$a[y_1(t) + y_2(t)] + (x_1(t) + x_2(t)) \quad \text{linear}$$

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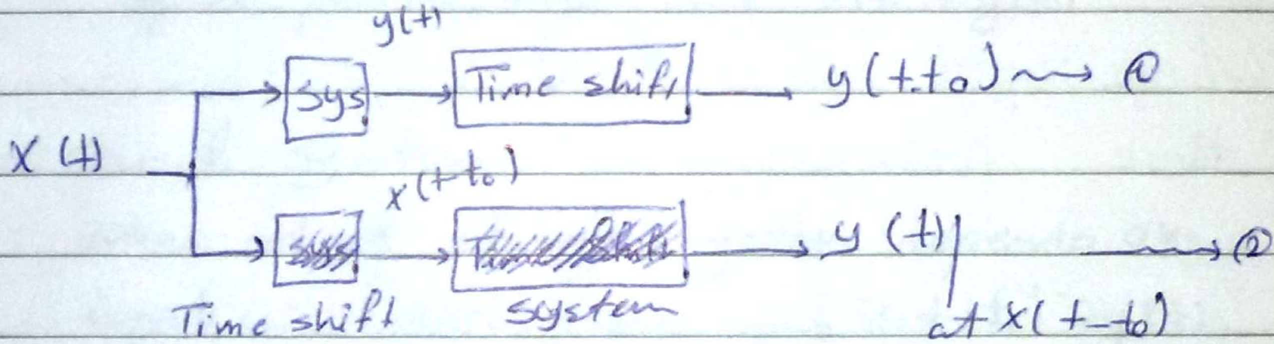
* Zero input zero output :- ZIZO

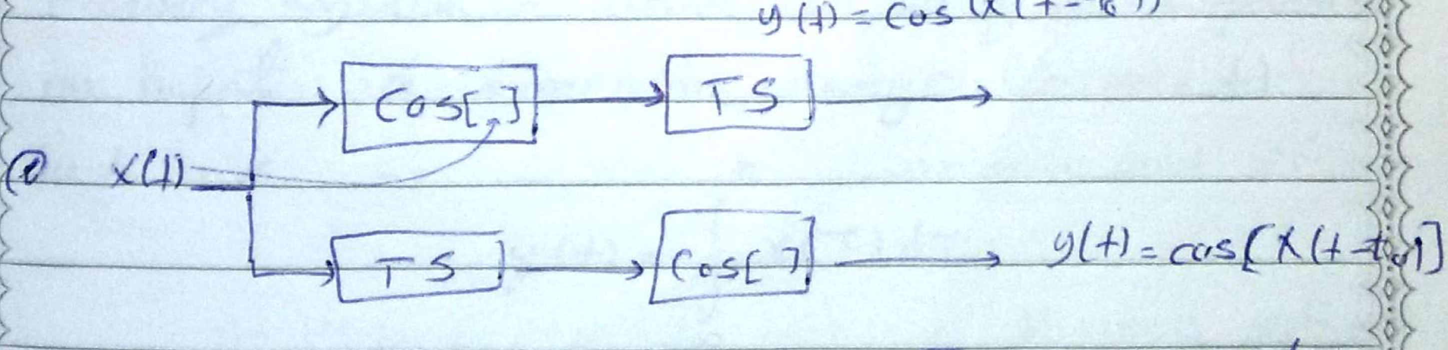
All linear systems are ZIZO system.
not All ZIZO systems are linear.

② Time variant / Invariant /



if ① = ② The system is Time invariant.
if ① ≠ ② " " " " variant

Ex) ① $y(t) = \cos[x(t)]$
② $y(t) = x(t)\cos(t)$



Time invariant

Subject: Time Variant أي t مع أي x حاجة يعني
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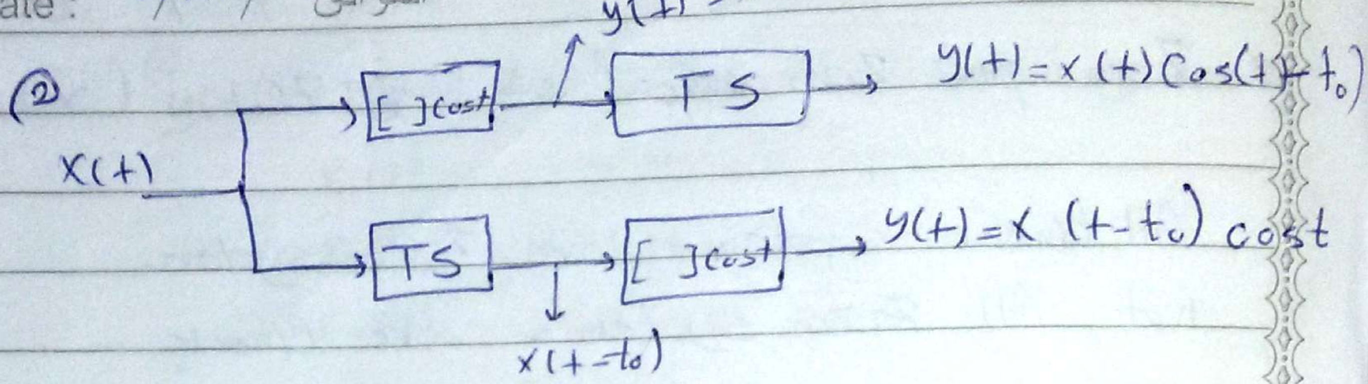
② 

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(4) Causel / non-Causal system

* Causal system :- output at the time (t_0) dosen't depend on input at times grater then (t_0).

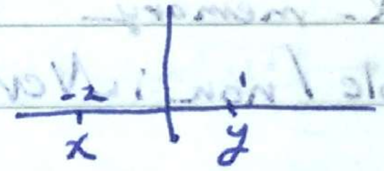
input past time $\longrightarrow x(t-t_0)=y(t)$

input present time $\longrightarrow x(t)=y(t)$

* non causal system :-

when output of any system depends on input in future time $\longrightarrow x(t+t_0)=y(t)$

Ex) $y(t)=x(t-3)$



(ε) memory / memoryless system

* memory system :- when output $y(t)$ depends on input at times in range $(-\infty, t)$

Such as =>

$$y(t)=\int_{-\infty}^{t} x(\tau)\,d\tau$$

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* also when input is $x(t+t_0)$ system a memory.

* memoryless system :-

All causal systems are memoryless system

when input and output happend in same time $x(t) = y(t)$

$y(t) = e^{-2t} x(t+4)$, $y(t) = 10\cos(t\, x(t))$

noncausal - memory   causel + memory less

⑥ Invertible / non invertible systems.

2 differnt inputs gives 2 differet outputs it's invertible.

2 different inputs gives 2 same outputs it's non invertible.

Any periodic system is non-invertable

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Ex).

① $y(t)=\int_{-\infty}^{t} x(t)\,dt$

↓ invertable

$x(t) \rightarrow \boxed{\int} \rightarrow y(t)$

$y(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow x(t)$

② $y(t)=\cos[x(t)]$

→ periodic & non invertable

③ $y(t)=x^2(t)$

$1 \longrightarrow \boxed{\square^2} \rightarrow 1$

non invertable.

$-1 \longrightarrow \boxed{\square^2} \rightarrow 1$

⑦ stable / unstable

↓ BIBO / ↓ not BIBO

BIBO "Bounded input Bounded output"

* The system is stable when it's output $\neq \infty$ at any given input...

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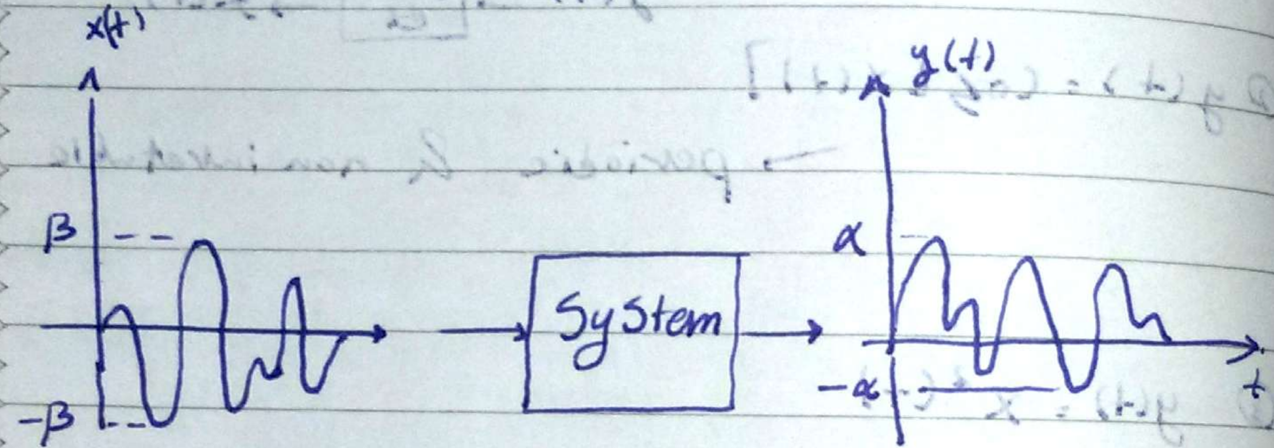
Ex) $y(t) = e^{x(t)}$ check system stability

replace $x(t) \xrightarrow{By} \beta$ where β is a constant number and check if $y(t) = \infty$?

$y(t) = e^{\beta} \neq \infty \Rightarrow$ system is stable

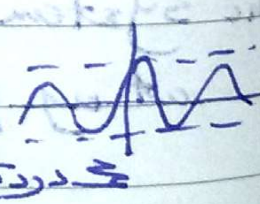


Ex) $y(t) = \int_{-\infty}^{t} x(t)\, dt \longrightarrow$ is it BIBO?

$= \int_{-\infty}^{t} \beta\, dt = \beta \int_{-\infty}^{t} dt = \beta\, (t)\Big|_{-\infty}^{t}$

$= \beta\, t + (-\infty) = \infty$ not stable

Ex) $y(t) = 10 \cos[t\, x(t)^2]$



cos & sin stable except tan.

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Representation of linear systems:-

① Integrator:-

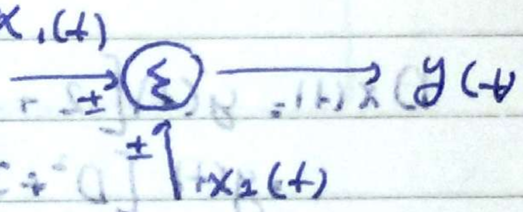
$$y(t) = \int x(t)\,dt \qquad x(t) \longrightarrow \underset{D^{-1}}{\boxed{\int}} \longrightarrow y(t)$$

② Differentiator:-

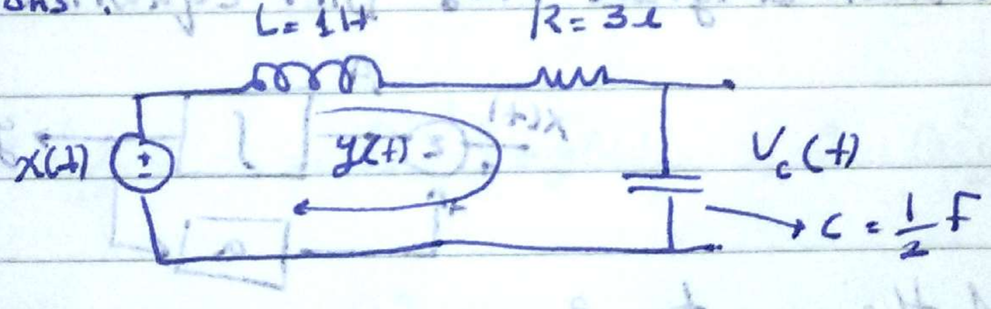
$$y(t) = \frac{\partial x(t)}{\partial t} \qquad x(t) \longrightarrow \underset{D}{\boxed{\frac{d}{dt}}} \longrightarrow y(t)$$

③ Adder:-

$$y(t) = x_1(t) \pm x_2(t)$$



Ex) Represent this system in differential Equations:



find the relation between input voltage $x(t)$ and $y(t) = i(t)$ as a current.

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$$x(t) = V_C(t) + V_L(t) + V_R(t)$$

$$x(t) = \frac{1}{C}\int_{-\infty}^{t} y(t)\,dt + L\frac{dy(t)}{dt} + Ry(t)$$

$$= 2\int_{-\infty}^{t} y(t)\,dt + 1\,\frac{dy(t)}{dt} + 3y(t)$$

$$= 2D^{-1}y(t) + Dy(t) + 3y(t)$$

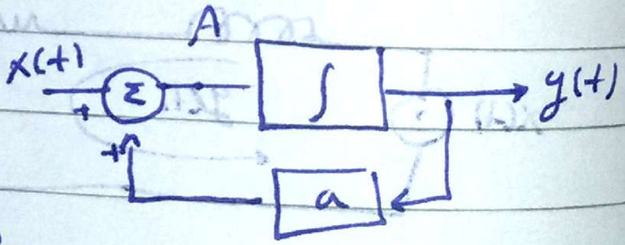
$$Dx(t) = 2y(t) + D^2y(t) + 3Dy(t)$$

$$Dx(t) = y(t)\left[2 + D^2 + 3D\right]$$

$$= y(t)\left[D^2 + 3D + 2\right]$$

* From block diagram find Diff equation :-

Ex)



find the system?

$A = x(t) + ay(t)$, $\int A = y(t)$

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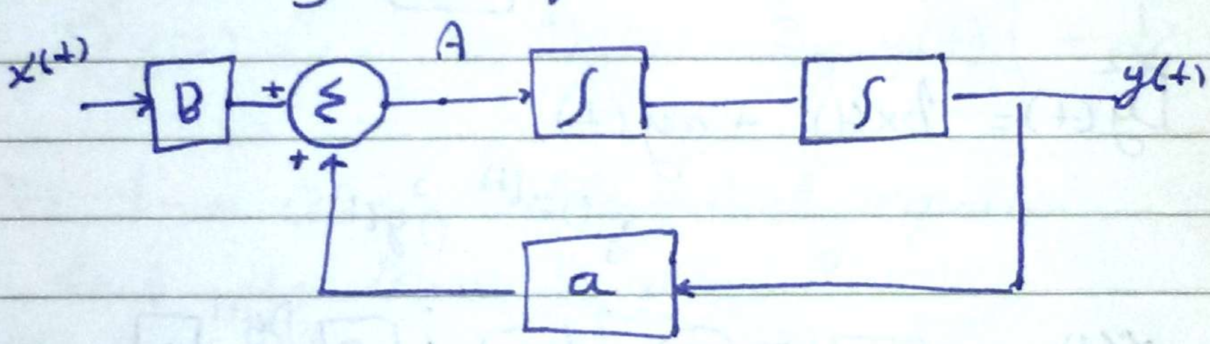
$\int [x(t) + a y(t)] = y(t)$

$x(t) + a y(t) = \frac{\partial y(t)}{\partial t} \Rightarrow x(t) + a y(t) = D y(t)$

* ميزة الـ feedback توضح مسار الـ system و y(t) لو اتقل على الطاير.

Ex) find system equation of these block diagram?



$A = B x(t) + a y(t)$

$\iint A = y(t) \Rightarrow D^{-2} A = y(t) \Rightarrow A = D^{2} y(t)$

$B x(t) + a y(t) = D^{2} y(t)$

$$\frac{a y(t) [D^{2} - a]}{B} = x(t)$$

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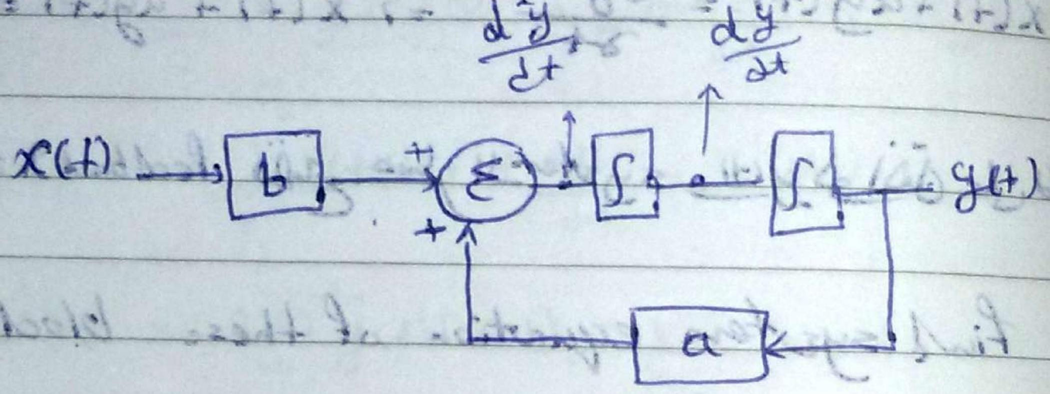
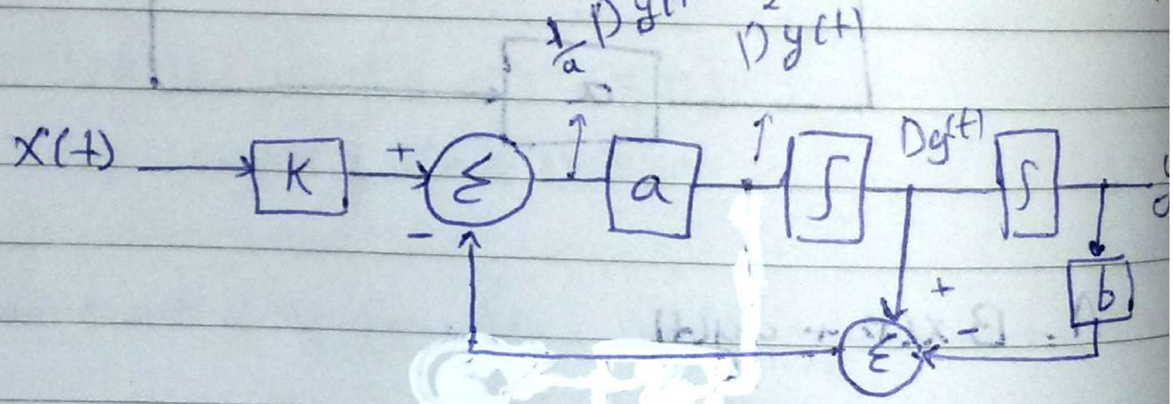
Q1- What is the system described by this block diagram?

(1)



$$D^2y(t) = bx(t) + ay(t)$$

(2)



$$D^2y(t) = Kx(t) - [Dy(t) - by(t)]$$

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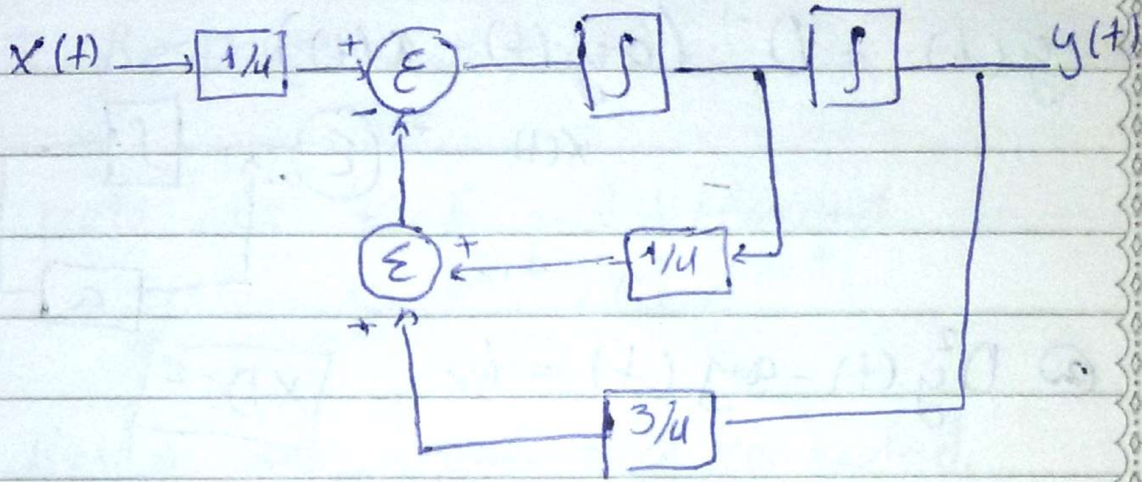
③



$$D^2 y(t) = -\left[\tfrac{1}{4} D y(t) + \tfrac{3}{4} y(t)\right] + \tfrac{1}{4} x(t)$$

Q2 from Given Differenitial equaiton find the Block diagram ?

① $\frac{d}{dt} y(t) - a y(t) = x(t)$

② $\frac{d^2}{dt^2} y(t) - a y(t) = b\, x(t)$

Solution

① $D y(t) - a y(t) = x(t) \quad \boxed{\times D^{-1}}$

$y(t) = D^{-1} a y(t) + D^{-1}(x(t))$

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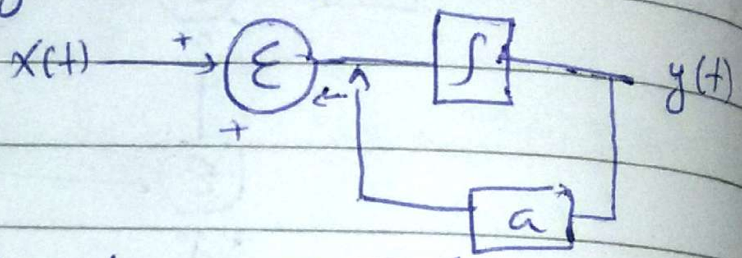
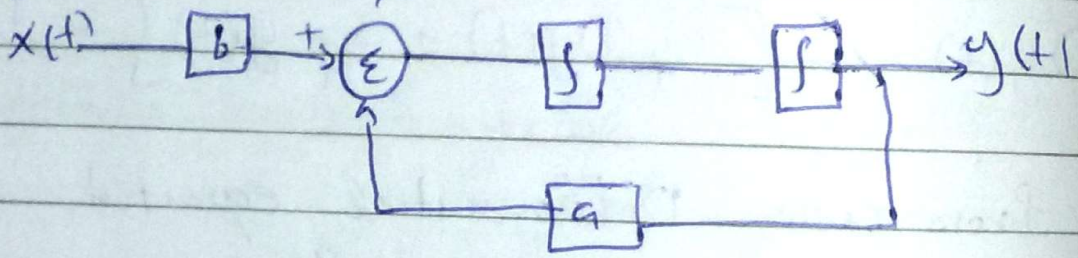
$$y(t) = D^{-1}\left(ay(t) + x(t)\right)$$



② $D^2 y(t) - ay(t) = bx$ $\boxed{\times D^{-2}}$

$$y(t) = D^{-2}\left[ay(t) + bx(t)\right]$$



ملاحظة/ لو مامش في المعادلة D مرة يعني مافيش feedback من الوسط.

ولو في المعادلة الأصلية y(t) مضروبة في عدد يعني فيها feedback .

Subject zero input موضوع الدرس
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* Z - I Response $\rightsquigarrow x(t) = 0$

① if roots are real and different.

$$y_0(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

② if Roots are real and repeated.

$$y_0 = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_1 t}$$

③ if Roots are complex

$$y_0(t) = C e^{\alpha t} \cos(\beta t + \theta)$$

example1 find Z-I Response for :-

① $D^2 y(t) + 3 D y(t) + 2 y(t) = D x(t) \Rightarrow y(0) = 0$
$y'(0) = -5$

$\lambda^2 + 3\lambda + 2 = 0$

$(\lambda + 1)(\lambda + 2) = 0$ $\boxed{\lambda_1 = -1}$ $\boxed{\lambda_2 = -2}$

$$y_0(t) = C_1 e^{-t} + C_2 e^{-2t}$$

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$y_o(t) = C_1 e^{-t} + C_2 e^{-2t} = 0$

$C_1 e^{0} + C_2 e^{0} = 0 \Rightarrow C_1 + C_2 = 0 \longrightarrow ①$

$\frac{dy(t)}{dt} = -C_1 e^{-t} - 2C_2 e^{-2t} = -5$

$-C_1 e^{0} - 2C_2 e^{0} = -5$

$-C_1 - 2C_2 = -5 \longrightarrow ②$

$\therefore y_o(t) = -5e^{-t} + 5e^{-2t}$

② $D^2 y(t) + 6Dy(t) + 9y(t) = 3Dx(t) + 5x(t)$

when $y(0) = 3$, $\dot{y}(0) = -7$

$\lambda^2 + 6\lambda + 9 = 0$

$(\lambda + 3)(\lambda + 3) = 0 \Rightarrow \lambda_1 = -3$, $\lambda_2 = -3$

$y_o(t) = C_1 e^{-3t} + C_2 t e^{-3t}$

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$$3 = C_1 e^{-3t} + \cancel{C_2 (0) e^{-3t}}$$

$$\Rightarrow C_1 e^{-3t} = 3 \Rightarrow \boxed{C_1 = 3} \longrightarrow ①$$

$$\frac{d}{dt}\downarrow \quad -7 = -3C_1 e^{-3(0)} + C_2 \left[-3 \cancel{t} e^{-3t} + e^{-3t} \right]$$

$$-7 = -9 + C_2 \Rightarrow \boxed{C_2 = 2}$$

$$\therefore y(t) = 3e^{-3t} + 2te^{-3t} \quad \#$$

③ $D^2 y(t) + 4 D y(t) + 40 y(t) = x(t)$
when $y(0) = 2$, $y'(0) = 16.78$

$$\lambda^2 + 4\lambda + 40 = 0$$
$$(\lambda + 2 - j6)(\lambda + 2 + j6) = 0 \qquad \alpha = -2$$
$$\beta = 6$$

$$y = Ce^{\alpha t} \cos(\beta t + \theta)$$
$$y = Ce^{-2t} \cos(6t + \theta)$$

$$2 = Ce^{-2(0)} \cos(6(0) + \theta)$$
$$2 = C \cos(\theta) \longrightarrow ①$$

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$16.78 = Ce^{-2t}\cdot(-6\sin(6t+\theta)) + (-2Ce^{-2t})\cdot$
$\cos(6t+\theta) \rightarrow ①$

$16.78 = -C\,6\sin\theta - 2\,C\cos.\theta \rightarrow 2$

$16.78 = -C\,6\sin\theta - 4$

$\frac{-20.78}{6} = C\sin\theta \rightarrow ③$

$\frac{C\sin\theta}{C\cos\theta} = \tan\theta = \frac{-3.46}{2} = -1.73$

$\theta = -59.92^\circ$

$C = \frac{2}{\cos(59.92)} = 4$

$\therefore y_\circ(t) = 4e^{-2t}\cos(6t + (-59.92))$

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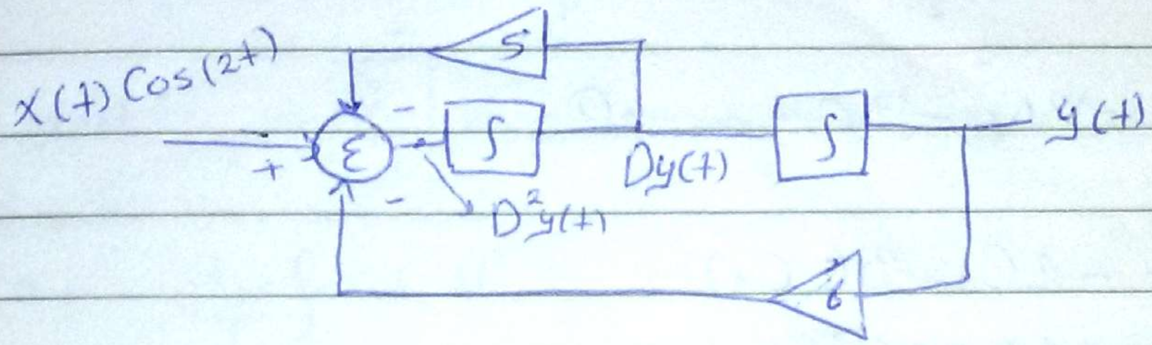
Example 1 — سؤال امتحان الثاني عام 2013



$$D^2y(t) = -5Dy(t) - 6y(t) + x(t)\cos(2t)$$

$$D^2y(t) + 5Dy(t) + 6y(t) = \cancel{x(t)\cos(2t)} \rightarrow 0$$

Z - I Response

$$\lambda^2 + 5\lambda + 6 = 0$$

$$(\lambda + 3)(\lambda + 2) = 0 \qquad \lambda = -3 , \lambda = -2$$

$$y_0(t) = C_1 e^{-2t} + C_2 e^{-3t}$$

when $y(0) = 1$

$$1 = C_1 + C_2 \longrightarrow ①$$

when $y'(0) = -4$

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$$-4 = -2C_1 e^{-2t} - 3C_2 e^{-3t}$$

$$-4 = -2C_1 - 3C_2 \rightsquigarrow (2)$$

$$-4 = -2C_1 - 3(1 - C_1)$$

$$\cancel{-4 = -2C_1 - 3 + 3C_1}$$

$$\boxed{-1 = C_1} \qquad \boxed{C_2 = 2}$$

$$\therefore\ y_o(t) = -1e^{-2t} + 2e^{-3t} \quad \#$$

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* Fourier series :- ① (Trigonometric form)

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

DC value

$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cos(n\omega_0 t)\, dt$$

$$b_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cdot \sin(n\omega_0 t)\, dt$$

② (Compact form)

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos[n\omega_0 t + \theta_n]$$

$$C_0 = a_0$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

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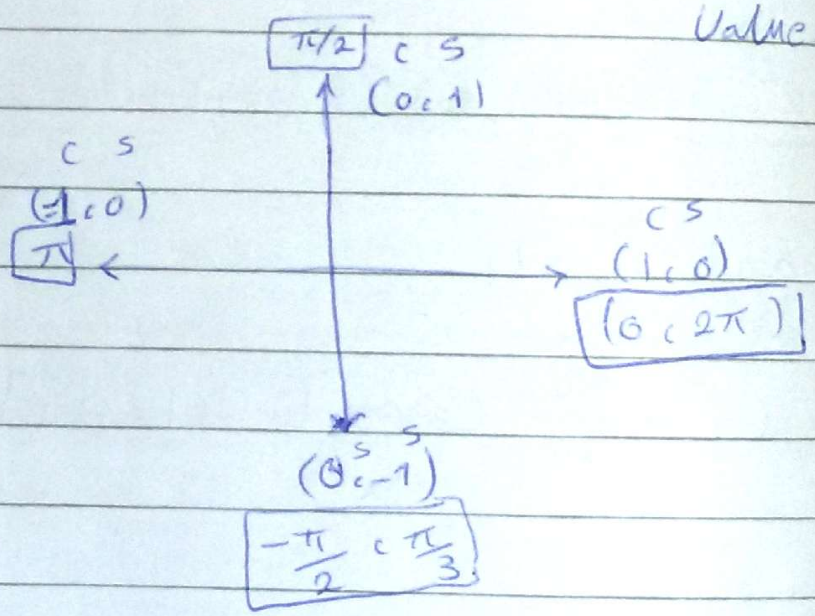
* Keep in mind:-

① $\int_0^{T_0} \sin(n\omega_0 t)\,dt = 0$ for all n value

② $\int_0^{T_0} \cos(n\omega_0 t)\,dt = 0$ for all value except n=0

③ $\int_0^{T_0} \sin(n\omega_0 t)\cos(m\omega_0 t)\,dt = 0$ for all n, m values



مكاملات :-

$\int x\cos x =$

$= \cos x + x\sin x$

$\int x\sin x =$

$= \sin x - x\cos x$

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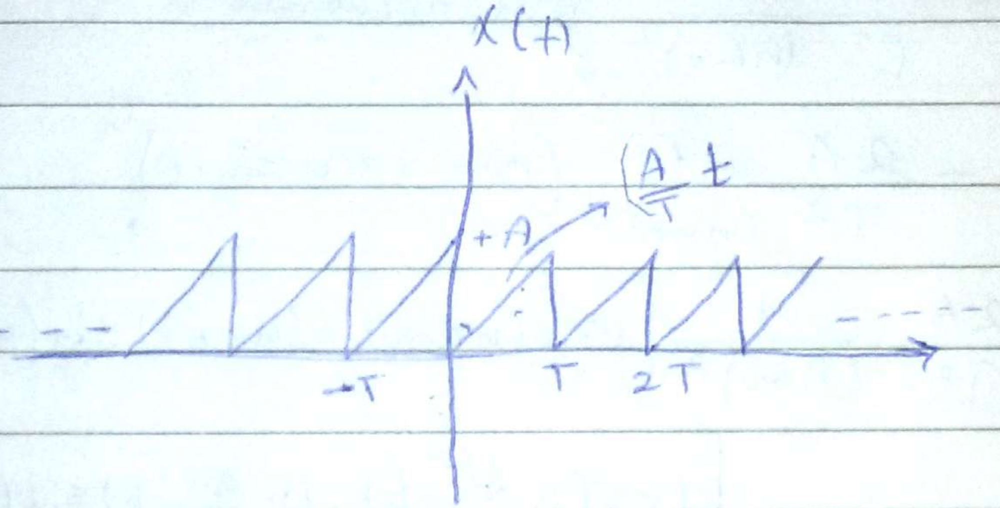
Find FS Coefficients for $x(t)$:-
a_0, a_n, b_n



$$a_0 = \frac{1}{T}\int_0^T \frac{A}{T}\, t\, dt$$

$$= \frac{A}{T^2}\int_0^T t\, dt \Rightarrow \frac{A}{T^2}\left(\frac{t^2}{2}\right)_0^T = \frac{A}{\cancel{T^2}}\,\frac{\cancel{T^2}}{2}$$

$$\boxed{= \frac{A}{2}}$$

$$a_n = \frac{2}{T}\int_0^T \frac{A}{T}\, t \cos(n\omega_0 t)\, dt$$

$$= \frac{2A}{T^2}\int_0^T t \cos(n\omega_0 t)\, dt$$

حل بالتعويض

$u = n\omega_0 t \longrightarrow t = \frac{u}{n\omega_0}$

$du = n\omega_0\, dt \longrightarrow dt = \frac{du}{n\omega_0}$

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$$= \frac{2A}{T^2} \int_0^T \frac{u}{n\omega_0} \cos(u) \frac{du}{n\omega_0}$$

$$= \frac{2A}{T^2} \frac{1}{(n\omega_0)^2} \int_0^T u \cos(u)\, du$$

$$= \frac{2A}{T^2} \frac{1}{(n\omega_0)^2} \left(\cos u + u \sin u\right)_0^{T_0}$$

$$= \frac{2A}{T^2} \frac{1}{(n\omega_0)^2} \left[\cos(n\omega_0 t) + (n\omega_0 t)\sin(n\omega_0 t)\right]_0^{T_0}$$

$$= \text{''} \quad \text{''} \quad \left[\cos\left(n\frac{2\pi}{T_0}t\right) + \left(n\frac{2\pi}{T_0}t\right)\sin\left(n\frac{2\pi}{T_0}t\right)\right]_0^{T_0}$$

$$= \text{''} \quad \text{''} \left[\cos\left(n\frac{2\pi}{\cancel{T_0}}\cancel{T_0}\right) + \left(n\frac{2\pi}{\cancel{T_0}}\cancel{T_0}\right)\sin\left(n\frac{2\pi}{\cancel{T_0}}\cancel{T_0}\right)\right] -$$

$$= \left[\underbrace{\cos 0}_{1} + 0\right]$$

$$= \text{''} \text{''} \left[\overset{1}{\cos n2\pi} + (n2\pi)\overset{0}{\sin(n2\pi)} - 1\right]$$

$$= 1 + 0 - 1 = \boxed{0}$$

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$$b_n = \frac{2}{T}\int_0^T \frac{A}{T}\, t \cdot \sin(n\omega_0 t)\, dt$$

$$= \frac{2A}{T^2}\int_0^T t \cdot \sin(n\omega_0 t)\, dt$$

$$u = n\omega_0 t \longrightarrow t = \frac{u}{n\omega_0}$$

$$du = n\omega_0\, dt \longrightarrow dt = \frac{du}{n\omega_0}$$

$$= \frac{2A}{T^2}\int_0^T \frac{u}{n\omega_0} \sin(u)\, \frac{du}{n\omega_0}$$

$$= \frac{2A}{T^2} \cdot \frac{1}{(n\omega_0)^2}\int_0^T u \sin u\, du$$

$$= \text{''} \quad \text{''} \quad \left[\sin u - u\cos u\right]_0^T$$

$$= \text{''} \quad \text{''} \left[\sin(n\omega_0 t) - (n\omega_0 t)\cos(n\omega_0 t)\right]_0^T$$

$$= \text{''} \quad \text{''} \left[\underbrace{\sin\left(n\frac{2\pi}{\cancel{T}}\cancel{T}\right)}_{0} - \left(n\frac{2\pi}{\cancel{T}}\cancel{T}\right)\underbrace{\cos\left(n\frac{2\pi}{\cancel{T}}\cancel{T}\right)}_{1}\right]$$

$$- \left[0 - \underbrace{0}_{0}\right]$$

$$= \frac{2A}{T^2} \cdot \frac{1}{(n\omega_0)^2} \cdot -n2\pi = -\frac{\cancel{2}A}{T^2 n^{\cancel{2}} \frac{4\pi^2}{T^2}} \cdot \cancel{n}\cancel{2}\pi = \boxed{-\frac{A}{n\pi}}$$

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$$\therefore \quad x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \left[0 + \frac{-A}{n\pi} \sin(n w_0 t) \right]$$

لا صفري

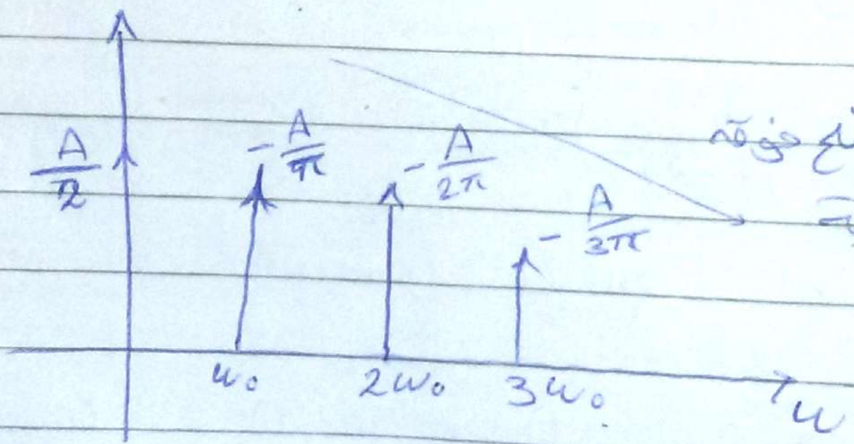
$$x(t) = \frac{A}{2} - \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n w_0 t)$$

ومادام السريسه = $\boxed{n=2}$ $n=1$

$$= \frac{A}{2} - \frac{A}{\pi} \sin(w_0 t) - \frac{A}{\pi} \cdot \frac{1}{2} \sin(2 w_0 t)$$

$n=3$

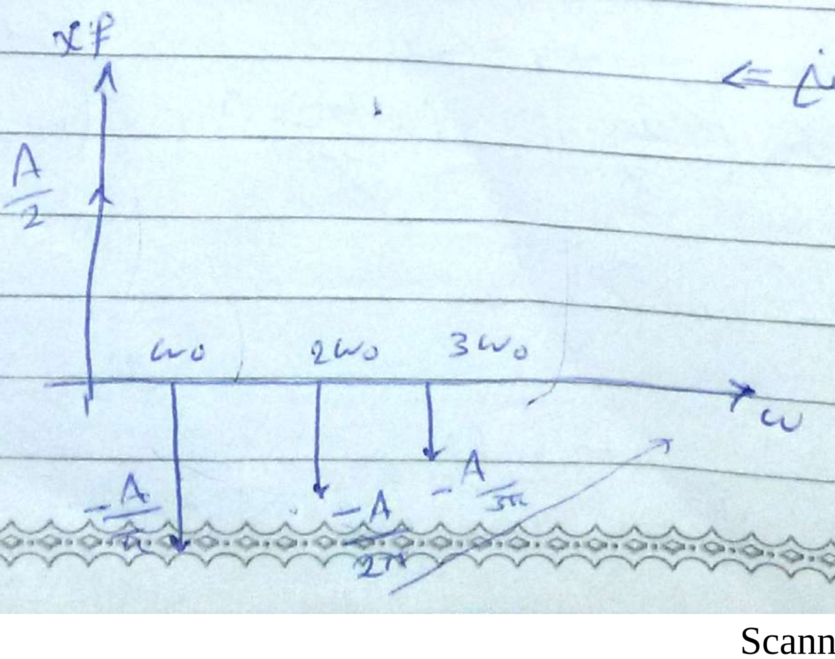
$$- \frac{A}{\pi} \frac{1}{3} \sin(3 w_0 t)$$



لتوقع فوقه تناقصية

$\infty = 0$

رسم $\Leftarrow$



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Fourier Series "Symmerty Conditions:-

① if $x(t)$ is a "Zero-mean" signal

then :- $\boxed{a_0=0}$

② if $x(t)$ is odd signal $[x(-t)=-x(t)]$

then: $\boxed{a_0=0}$ & $\boxed{a_n=0}$

$$b_n=\frac{4}{T_0}\int_0^{T_0/2} x(t).\sin(n\omega_0 t)$$

③ if $x(t)$ is even Signal $[x(-t)=x(t)]$

then :- $\boxed{b_n=0}$

$$\boxed{a_0=\frac{2}{T_0}\int_0^{T_0/2} x(t).dt}$$

$$\boxed{a_n=\frac{4}{T_0}\int_0^{T_0/2} x(t).\cos(n\omega_0 t)\,dt}$$

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④ "Half - wave" Symmetry $\rightarrow \left[x(t) = -x\left(t \pm \frac{T}{2}\right)\right]$

Then :- $\boxed{a_0 = 0}$

The series will consist only of (odd) harmonics and all Even harmonic are (Zero)

$\boxed{a_n = b_n = 0}$ $\xrightarrow{\text{only}}$ for (n) Even .

for "odd" haronics you should find an & bn as:-

$$a_n = \frac{4}{T}\int_0^{T/2} x(t).\cos(n\omega_0 t)\, dt$$

$$b_n = \frac{4}{T}\int_0^{T_0/2} x(t).\sin(n\omega_0 t)\, dt$$

ملاحظة / مرات بيقولك إحسب أول 9 حدود وواضحة
بيووقت هنا فا أكيد تلقائي Half wave باشـ
الحدود الزوجية = 0 يعني ←

1 ② 3 ④ 5 ⑥ 7 ⑧ 9

0 0 0 0

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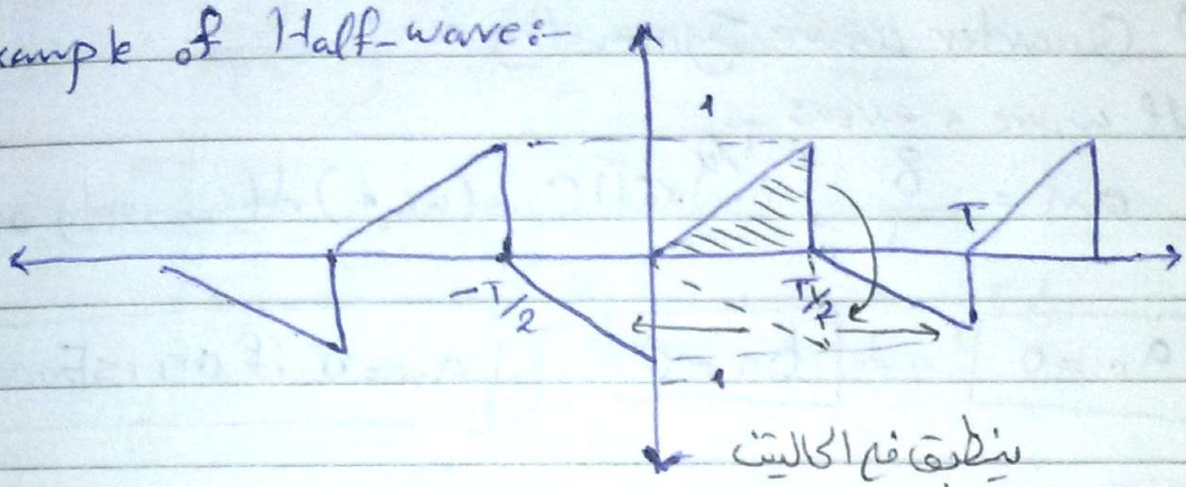
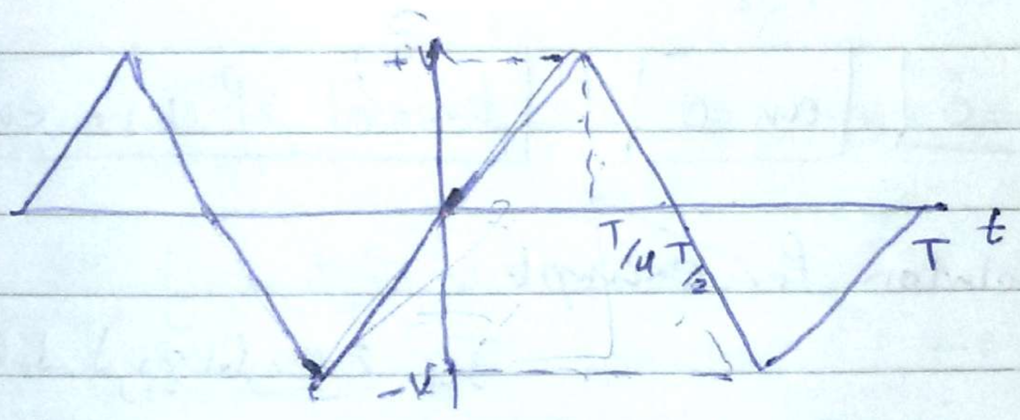
* Example of Half-wave:-



fourier serien Time continue

* Find FSTC of the given Signal $x(t)$?



$a_0 = 0$

$a_n = 0$

$b_n = \checkmark$

هادي مش Q - Wave symetry

Half w + odd $\rightarrow b_n = \frac{8}{T_0}\int_0^{T/4} x(t)\sin(n\omega_0 t)\,dt$

Half w + even $\rightarrow a_n = \frac{8}{T_0}\int_0^{T/4} x(t)\cos(n\omega_0 t)\,dt$

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(5) Quarter wave Symmetry :-

(A) half wave + even :

$$a_n = \frac{8}{T}\int_0^{T/4} x(t)\cos(n\omega_o t)\,dt \longrightarrow \text{only odd } n$$

$\boxed{a_o = 0}$ $\boxed{b_n = 0}$ $\boxed{a_n = 0 \text{ if } (n) \text{ is Even}}$

(B) Half wave + odd :

$$b_n = \frac{8}{T}\int_0^{T/4} x(t).\sin(n\omega_o t)\,dt \longrightarrow \text{only odd } (n)$$

$\boxed{a_o = 0}$ $\boxed{a_n = 0}$ $\boxed{b_n = 0 \text{ if } (n) \text{ is even}}$

* solution for example :-

نكامل لربع الموجة $T/4$

$$\therefore b_n = \frac{8}{T_o}\int_0^{T_o/4} \frac{4V}{T_o}\, t.\sin(n\omega_o t)\,dt$$

$$\frac{32\,V}{T_o^2}\int_0^{T/4} t\sin(n\omega_o t)\,dt$$

$u = n\omega_o t \longrightarrow t = \frac{u}{n\omega_o}$

$du = n\omega_o\,dt \longrightarrow dt = \frac{du}{n\omega_o}$

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$$= \frac{32V}{T_0^2}\int_0^{T/4} \frac{u}{n\omega_0}\sin(u)\frac{du}{n\omega_0}$$

$$= \frac{32V}{T^2}\cdot\frac{1}{(n\omega_0)^2}\int_0^{T/4} u\sin u\,du$$

$$= \text{〃} \cdot \text{〃} \Big[\sin u - u\cos u\Big]_0^{T/4}$$

$$= \text{〃} \quad \text{〃} \Big[\sin(n\omega_0 t) - (n\omega_0 t)\cos(n\omega_0 t)\Big]_0^{T/4}$$

$$= \text{〃} \quad \text{〃} \Big[\sin\big(n\frac{2\pi}{T}\frac{T}{4}\big) - \big(n\frac{2\pi}{T}\cdot\frac{T}{4}\big)\cos\big(n\frac{2\pi}{T}\frac{T}{4}\big)\Big] - \boxed{0}$$

$$= \frac{\overset{8}{\cancel{32}}}{\cancel{T^2}}\cdot\frac{\cancel{T^2}}{n^2 \cancel{4}\pi^2}\Big[\sin\frac{n\pi}{2} - \frac{n\pi}{2}\,\overset{0}{\cancel{\cos\frac{n\pi}{2}}}\Big]$$

$$b_n = \frac{8V}{n^2\pi^2}\Big[\sin\frac{n\pi}{2}\Big]$$

ملاحظة :-

نفرضه فهو برقم ثابت لما كل ما نغير n يعطي نفس الرقم

لكن لو لكل n يطلع رقم مختلف معناها تتعدد الدالة

وتتحلل في السيريس.

$$x(t) = \underbrace{\frac{8V}{\pi^2}\left(\frac{1}{1}\right)\overset{1}{\cancel{\sin\frac{(1)\pi}{2}}}\sin(\omega t)}_{\text{1'st harmonic}} + \underbrace{\frac{8V}{\pi^2}\left(\frac{1}{9}\right)\underset{-1}{\cancel{\sin\frac{3\pi}{2}}}\sin(3\omega t)}_{\text{3'rd harmonic}}$$

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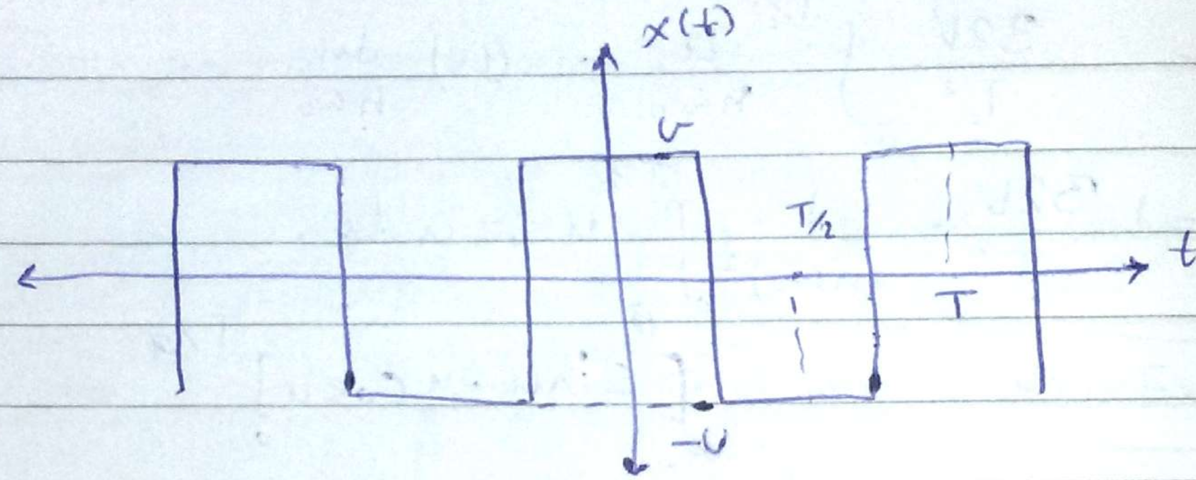

$a_0 = 0$

Even ✓

Half ✓ $\Rightarrow$ $b_n = 0$

a_n ✓ $\longrightarrow$ odd n

$$= \frac{8}{T_0}\int_0^{T/4} V\cos(n\omega_0 t)\,dt$$

$$= \frac{8V}{T_0}\cdot\frac{1}{n\omega_0}\Big[\sin(n\omega_0 t)\Big]_0^{T/4}$$

$$= \frac{8V}{T_0 n\omega_0}\left[\sin\left(n\frac{\cancel{2}\pi}{\cancel{T}}\frac{\cancel{T}}{\cancel{4}_{2}}\right) - 0\right]$$

$$= \frac{\overset{4}{\cancel{8}}V}{\cancel{T_0}\, n\, \cancel{2}\pi}\,\cancel{T_0}\,\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{4V}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

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Q. Calculate FS TC for this signal

$$x(t) = 3 + \cos(4t + \frac{\pi}{4}) + \sin(10t + \pi/3)$$

$DC = a_0 = 3$. $\omega_1 = \frac{2\pi}{T_1} = 4$ $\omega_2 = \frac{2\pi}{T_2} = 10$

$$T_0 = \pi$$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$x(t) = \cos(4t)\cos(\underbrace{\pi/4}_{\frac{1}{\sqrt{2}}}) - \sin(4t)\sin(\underbrace{\pi/4}_{\frac{1}{\sqrt{2}}}) + \sin(10t)\cos(\underbrace{\pi/3}_{\frac{1}{2}}) + \cos(10t)\sin(\underbrace{\pi/3}_{\frac{\sqrt{3}}{2}})$$

$$= 3 + \underbrace{\frac{1}{\sqrt{2}}\cos 4t - \frac{1}{\sqrt{2}}\sin 4t}_{n=2} + \underbrace{\frac{1}{2}\sin 10t + \frac{\sqrt{3}}{2}\cos 10t}_{n=5}$$

$\omega_0 = 2$

$n = 2$ $a_0 = 3$ $b_2 = -1/\sqrt{2}$ $a_3 = b_3 = 0$

$n = 5$ $a_1 = b_1 = 0$ $b_5 = \frac{1}{2}$ $a_4 = b_4 = 0$

$a_2 = \frac{1}{\sqrt{2}}$ $a_5 = \frac{\sqrt{3}}{2}$

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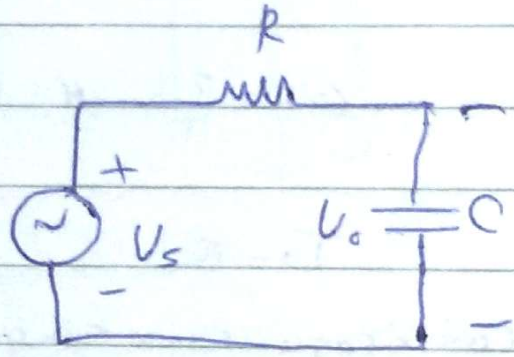
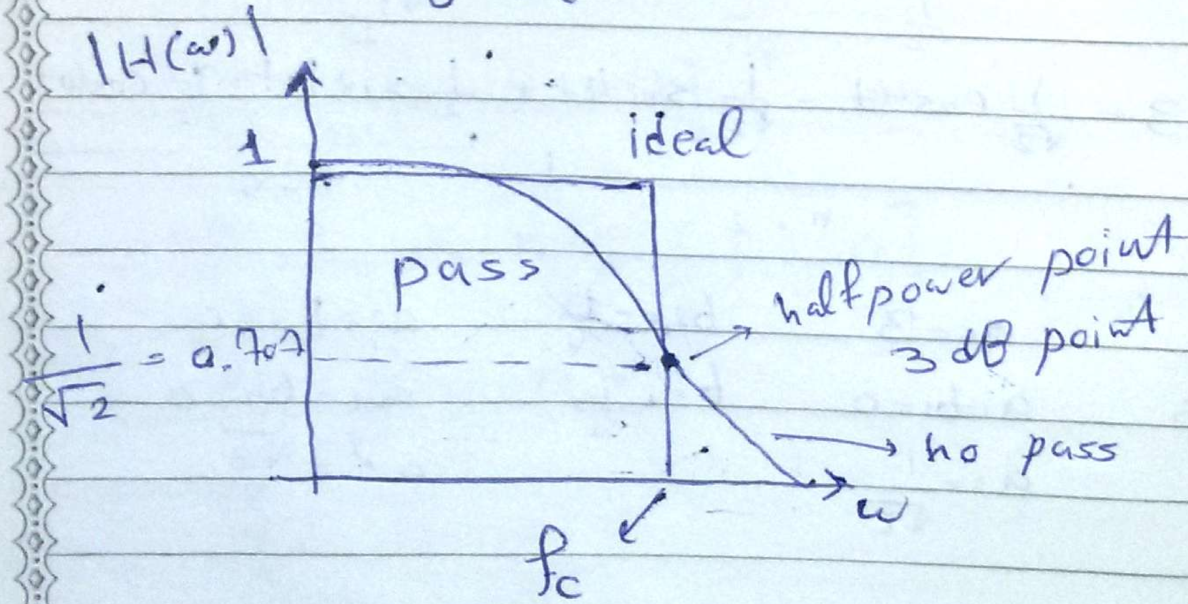
* Filters:-

① Low pass filtter (LPF)



$$H(\omega) = \frac{V_o}{V_i} = \frac{\cancel{I}\left(\frac{1}{j\omega C}\right)}{\cancel{I}\left(R + \frac{1}{j\omega C}\right)}$$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$



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$$3\,dB = 20 \log_{10}\left(\frac{1}{\sqrt{2}}\right)$$

$$|H(\omega_c)| = 0.707\, H_{max}$$

$$|H(\omega_c)| = \frac{1^2}{\sqrt{1+\omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}} \quad \boxed{\omega_c = \frac{1}{R}}$$

Note:- Decibels $\equiv$ dB is used to describe filter magnitudes.

$$dB = 20 \log_{10}(\text{level})$$

$$|H(\omega_c)|_{dB} = \left|\frac{1}{\sqrt{2}} H_{max}\right|_{dB} = 20 \log_{10}(H_{max})$$

$$+\, 20 \log_{10}\left(\frac{1}{\sqrt{2}}\right) = |H_{max}|_{dB} - 3$$

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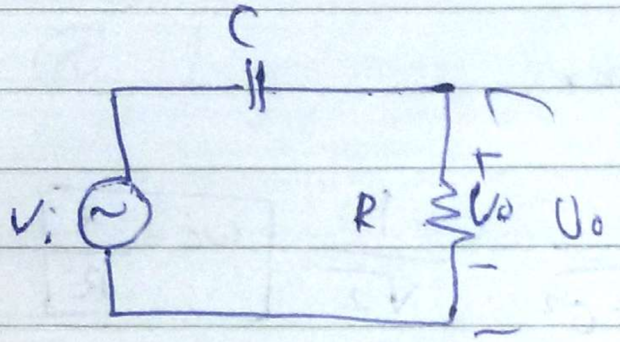
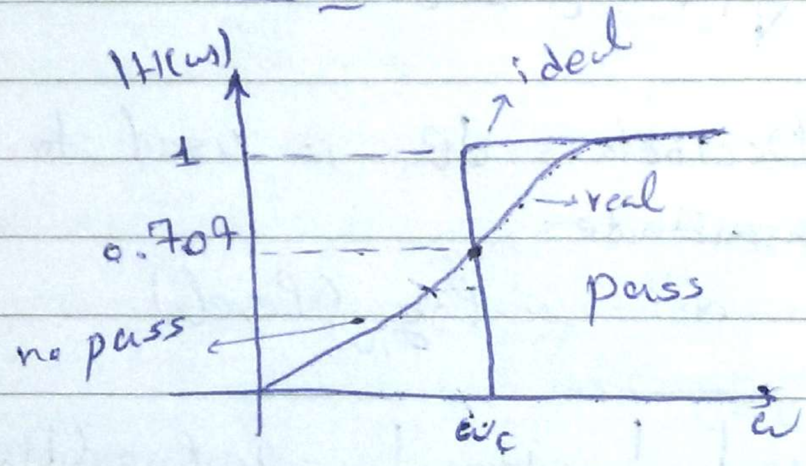
② high pass filter (HPF)



Vo

$$H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{j\omega C}}$$



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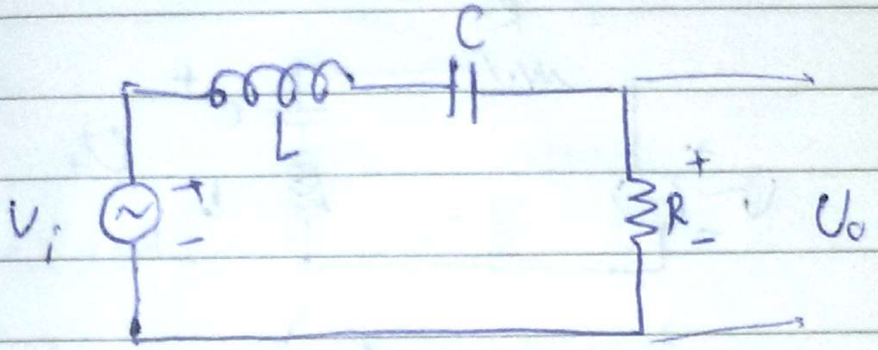
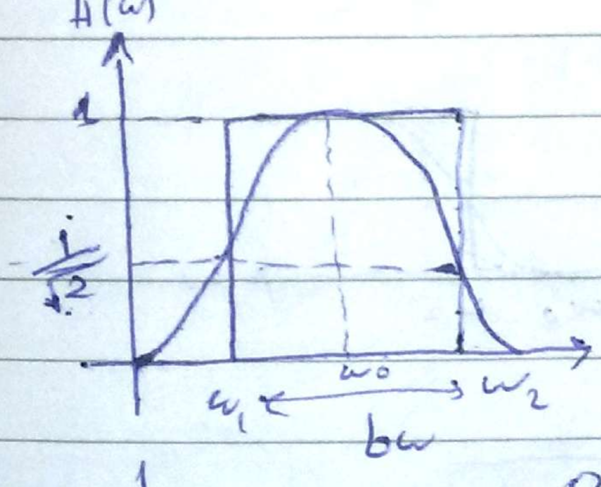
Band pass filter (BPF)



$$H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$BW = \omega_2 - \omega_1$$

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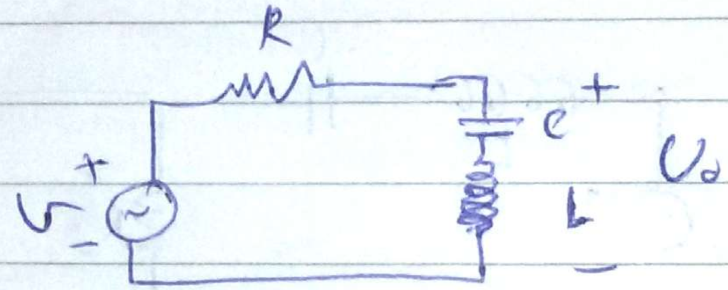
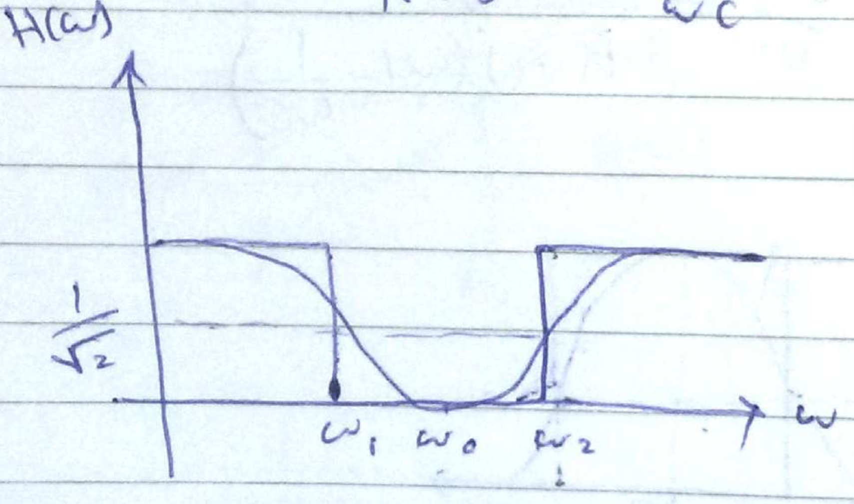
~~Band~~ Stop rejection filter (BRF)



$$H(\omega) = \frac{V_o}{V_i} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$



$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{LC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{LC}\right)^2 + \frac{1}{LC}}$$

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* Fourer Series [exponential form]

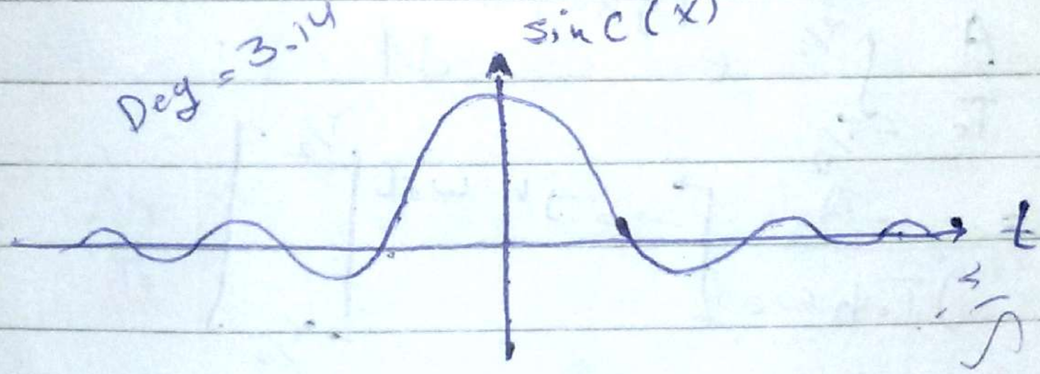
Recall:-

① $\cos(n\omega t) = \dfrac{e^{jn\omega t} + e^{-jn\omega t}}{2}$

② $\sin(n\omega t) = \dfrac{e^{jn\omega t} - e^{-jn\omega t}}{2j}$

③ $\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$ → Rad = 180 → $\text{sinc}(\pi x) = \dfrac{\sin(x)}{x}$

Deg = 3.14



$$x(t) = \sum_{n=-\infty}^{+\infty} D_n \, e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0} \int_0^{T_0} x(t) \, e^{-jn\omega_0 t} \, dt$$

$$D_0 = a_0 = \frac{1}{T_0} \int_0^{T_0} x(t) \, dt$$

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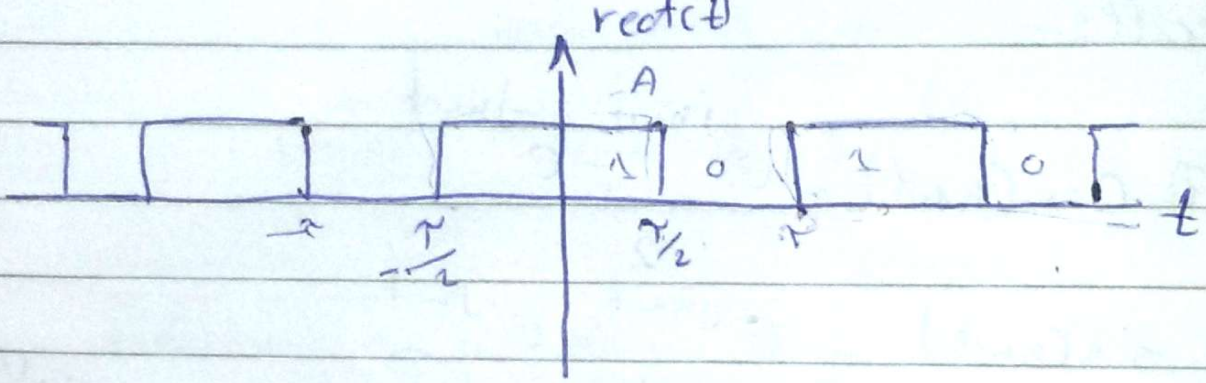
Example / using Exponention form

find F.S for ~~ag~~ given x(t)



$$D_n = \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A\, e^{-jnwt} . dt$$

$$D_n = \frac{A}{T_0}\int_{-\tau/2}^{\tau/2} e^{(-jnw)t} . dt$$

$$= \frac{-A}{jT_0 n w_0}\left[e^{-jnw_0 t}\Big|_{-\tau/2}^{\tau/2}\right]$$

$$= \frac{-A}{jT_0 n w_0}\left[e^{-jnw_0 \tau/2} - e^{jnw_0 \tau/2}\right]$$

$$= \frac{2A}{T_0 n w_0}\left[\frac{e^{jnw_0 \tau/2} - e^{-jnw_0 \tau/2}}{j2}\right]$$

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$$= \frac{2A}{T_0 n \omega_0} \sin\left(n\omega_0 \frac{\tau}{2}\right)$$

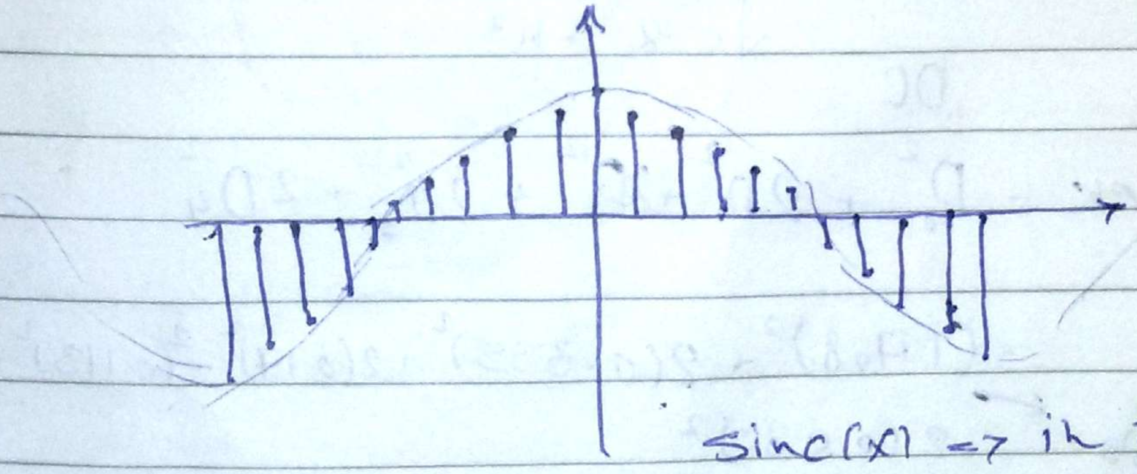
$$= \frac{\cancel{2}A}{\cancel{T_0} n \frac{2\pi}{\cancel{T_0}}} \sin\left(n \frac{2\pi}{T_0} \cdot \frac{\tau}{2}\right)$$

$$= \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right)$$

$$= \frac{A\tau}{T_0} \cdot \frac{\sin\left(\frac{n\pi\tau}{T_0}\right)}{\left(\frac{n\pi\tau}{T_0}\right)}$$

$$D_n = \frac{A\tau}{T_0} \sin\left(\frac{n\tau}{T_0}\right)$$

$$\frac{A\tau}{T_0} \sum_{-\infty}^{\infty} \operatorname{sinc}\left(\frac{n\tau}{T_0}\right) e^{jn\omega}$$



sinc(x) => in frequency domain

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* Power form exponential FS:-

$$Power = |D_0|^2 + 2\sum_{n=1}^{\infty} |D_n|^2$$

Given

$$D_n = \frac{0.3146}{0.2+jn}$$

* find the average power of this series.

Solution

$$|D_n| = \frac{0.3146}{\sqrt{0.2^2+n^2}}$$

DC

$$Power = D_0^2 + 2D_1^2 + 2D_2^2 + 2D_3^2 + 2D_4^2$$

$$= (1.708)^2 + 2(0.335)^2 + 2(0.17)^2 + 2(0.113)^2 + 2(0.085)^2$$

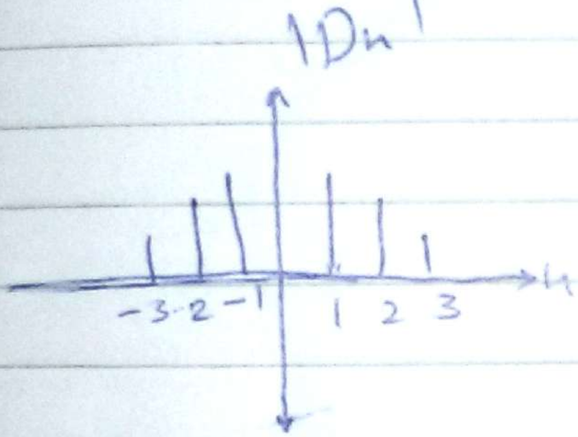
DC
مابضربناش
2 في

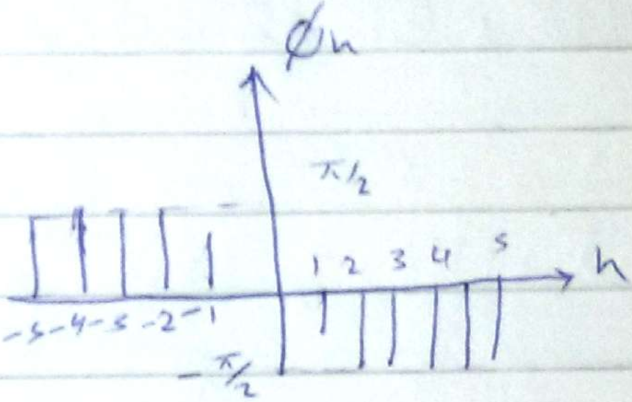
*Foruer spectrom :-

$D_n = |D_n| e^{j\phi_n} \rightarrow$ phase spectrom (odd)

$\downarrow$

amlitude spectrom (even)





Ex/ $D_n = \frac{0.3416}{0.2 + jn}$

$|D_n| = \frac{\sqrt{0.3416^2}}{\sqrt{0.2^2 + jn^2}}$

$\phi_n = 0 - \tan^{-1} 5n$

$= -\tan^{-1} 5n$

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مراجعة

* $D^2 y(t) + 3D y(t) + 2y(t) = D\, x(t)$

when $y(0) = 0$ $\quad y'(0) = -5$

الحالة :

$(D^2 + 3D + 2)y(t) = 0$ $\qquad$ Z-I

$\lambda^2 + 3\lambda + 2 = 0$

$(\lambda_1 + 1)(\lambda_2 + 2) = 0 \Rightarrow \boxed{\lambda_1 = -1} \quad \boxed{\lambda_2 = -2}$

$y_0(t) = C_1 e^{-\cancel{3}t} + C_2 e^{-2t}$

$\because y(0) = 0 \Rightarrow 0 = C_1 + C_2 \Rightarrow \boxed{C_1 = -C_2}$

$-5 = -C_1 - 2C_2$

$-5 = C_2 - 2C_2$

$-5 = -C_2 \Rightarrow \boxed{C_2 = 5} \qquad \boxed{C_1 = -5}$

$y_0(t) = -5e^{-t} + 5e^{-2t}$ ##

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* $D^2y(t) + 6Dy(t) + 9y(t) = 3Dx(t) + 5x(t)$

when $y(0) = 3$
$y'(0) = -7$

الحل

$\lambda^2 + 6\lambda + 9 = 0$

$(\lambda + 3)(\lambda + 3) = 0 \qquad \lambda_1 = -3 \qquad \lambda_2 = -3$

$y_0(t) = C_1 e^{-3t} + C_2 t e^{-3t}$

$\boxed{3 = C_1}$

$-7 = -3C_1 e^{-3t} - 3C_2 t e^{-3t} + C_2 e^{-3t}$

$-7 = -3C_1 + C_2$

$-7 = -3(3) + C_2 \qquad \boxed{C_2 = 2}$

$y_0(t) = 3e^{-3t} + 2te^{-3t}$ #

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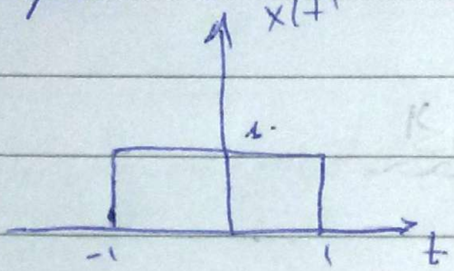
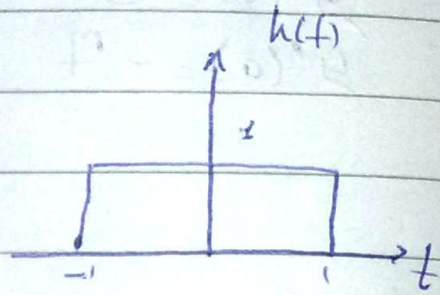
Graphical Convolution :-

Q2. Example /





Find :- $y(t) = x(t) * h(t)$

Solution :- $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$$h(t) = \begin{cases} 1 & -1 < t < 1 \\ 0 & e.w \end{cases}$$

$$h(\tau) = \begin{cases} 1 & -1 < \tau < 1 \\ 0 & e.w \end{cases}$$

$$h(t-\tau) = \begin{cases} 1 & -1 < t-\tau < 1 \\ 0 & e.w \end{cases}$$

$$h(t-\tau) = \begin{cases} 1 & t-1 < \tau < t+1 \\ 0 & e.w \end{cases}$$

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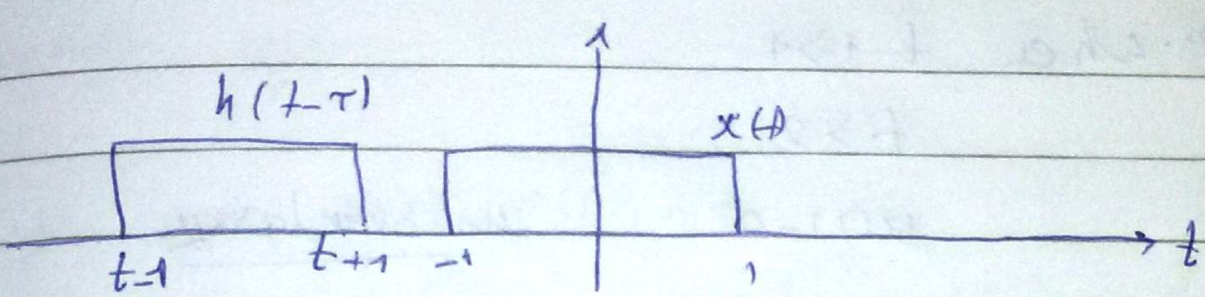
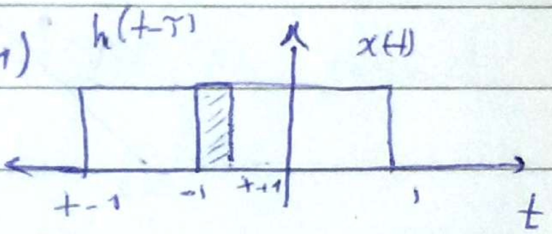


① when $\cancel{0}^{-1} < t+1 < 0$

$-2 < t < \cancel{0}\ -1$

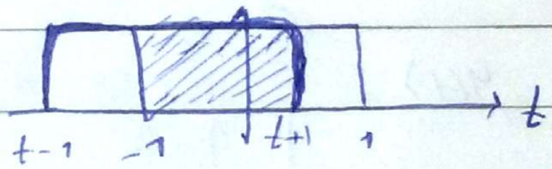
$y(t) = \int_{-1}^{t+1} (1).(1)\, d\tau = (t+1)-(-1)$

$= t+2$



② when :- $0 < t+1 < 1$

$-1 < t < 0$

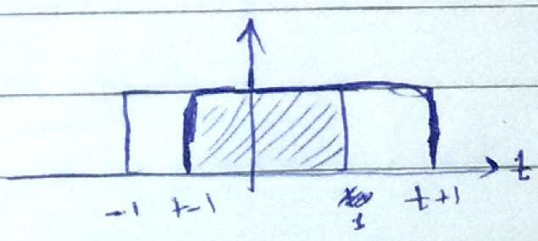


$y(t) = \int_{-1}^{t+1} (1)(1)\, d\tau = (t+1)-(-1)$

$= t+2$

③ When:- $-1 < t-1 < 1$

$0 < t < 2$



$y(t) = \int_{t-1}^{1} (1).(1)\, d\tau = 1-t-1$

$= -t+2$

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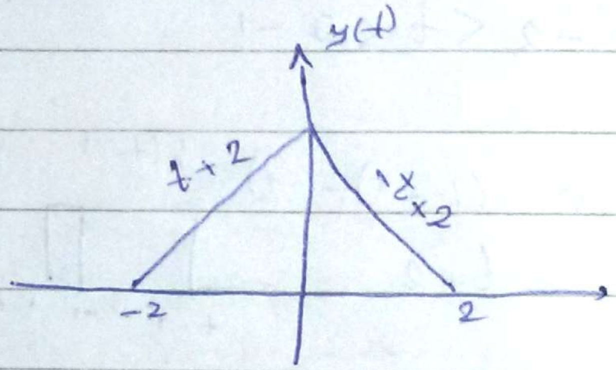
④ when $t-1>1$

$t>2$

$y(t)=0$ no overlaping

Total output $y(t)$:-



$$y(t)=\begin{cases} 0 & t<-2 \\ t+2 & -2\leq t<0 \\ -t+2 & 0\leq t<2 \\ 0 & t>2 \end{cases}$$

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test ② 2013

Q2- Given $x(t) = \cos 2\pi t$

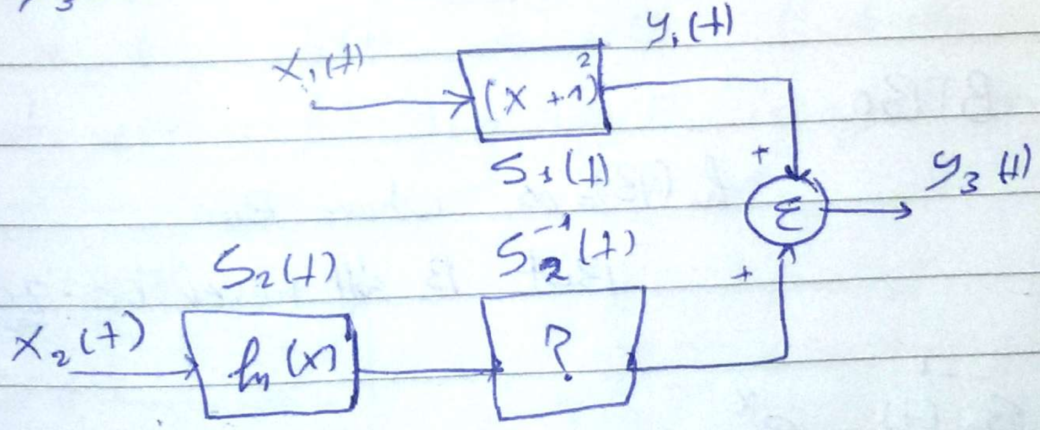
$x_2(t) = |\sin 2\pi t|^2 \rightarrow$ where $\sin 2\pi t \neq 0$

① Chick linearity of $S_2(t)$

② is $S_2(t)$ ZiZo ? is it BIBO? prove it

③ find $S_2^{-1}(t)$

④ find $y_3(t)$? is it periodic?



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Solution :-

① $x_1(t) \rightsquigarrow y_1 = \ln(x_1(t))$

$x_2(t) \rightsquigarrow y_2 = \ln(x_2(t))$

$x_3(t) \rightsquigarrow y_3 = \ln(x_3(t))$

$x_1 + x_2$ $\qquad$ $x_1 + x_2$

$\ln(x_1(t)) + \ln(x_2(t)) \neq \ln(x_1(t) + x_2(t))$

non linear #

② ZIZO عشان شرط ال $x(0)$ مش حتكون 0 يعني مش

ولو كانت 0 not ZIZO

$\ln 0 = \infty$

③ BIBO

$\ln(B) = \infty$ where $B = 0$

But B will never be zero

④ $S_2^{-1}(t) = e^x$

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(4) $y_3(t) = y_1 + y_2$

$y_1 = (\cos 2\pi t + 1)^2$

$y_2 = x_2 = (\sin(2\pi t))^2$

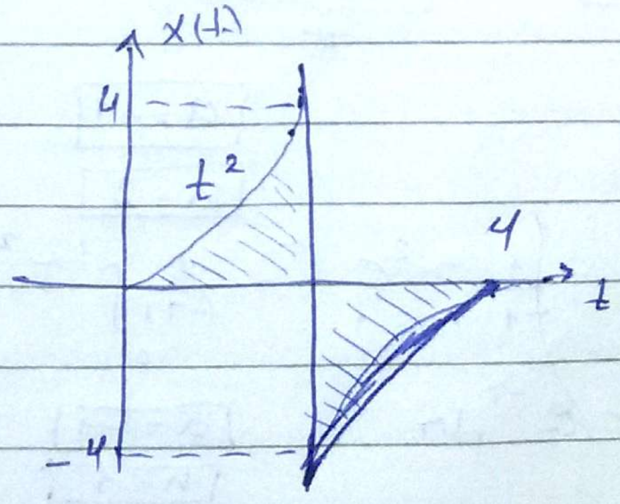
$y_3(t) = \cos^2(2\pi t) + 2\cos(2\pi t) + 1 + \sin^2(2\pi t)$

$\because \sin^2(x) + \cos^2(x) = 1$

$\therefore y_3(t) = 2\cos(2\pi t) + 2$

periodic

Q3- Considir the impulse Response of a systems $h(t) = e^{t} u(t)$ find it's output $y(t)$ if the input is given as:-



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$$y(t) = x(t) * h(t)$$

$$y(t) = [x_1(t) + x_2(t)] * h(t)$$

solution:

$$x(t) = \begin{cases} t^2 & 0<t<2 \\ \boxed{2t-8} & 2<t<4 \end{cases}$$

Recall:-

$$\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} \left[\frac{n}{a} \int x^{n-1} e^{ax} dx \right]$$

$$y(t) = \underbrace{x_1(t) * h(t)}_{\text{I}} + \underbrace{x_2(t) * h(t)}_{\text{II}}$$

① $\int_0^2 \tau^2 e^{(t-\tau)} d\tau$ $\boxed{a=-1}$ $\boxed{n=2}$

$$= \boxed{e^t} \int_0^2 \tau^2 e^{-\tau} d\tau = \frac{1}{-1} \tau^2 e^{-\tau} - \frac{2}{(-1)} \int_0^2 \tau^{2-1} e^{-\tau} d\tau$$

$$= -\tau^2 e^{-\tau} + 2\int_0^2 \tau e^{-\tau} d\tau$$ $\boxed{a=-1}$ $\boxed{n=1}$

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$$= \frac{1}{-1}\tau e^{-\tau} - \frac{1}{(-1)}\int_0^2 \overset{1}{\cancel{\tau^2}}\, e^{-\tau}\,d\tau$$

$$= -\tau^2 e^{-\tau} + 2\left[-\tau e^{-\tau} - e^{-\tau}\right]$$

$$= -\tau^2 e^{-\tau} - 2\tau e^{-\tau} - 2e^{-\tau}\Big|_0^2$$

$$= \left[-4e^{-2} - 4e^{-2} - 2e^{-2}\right] - \left[-2\right]$$

$$= \boxed{e^{t}}\left(-10e^{-2} + 4\right)$$

(II) $\rightarrow \int_2^4 [2\tau - 8]\, e^{(t-\tau)}\, d\tau = \int_2^4 2\tau e^{t-\tau}\, d\tau - 8\int_2^4 e^{t} e^{-\tau} d\tau$

$$= 2e^{t}\int_2^4 \tau e^{-\tau}\, d\tau - 8e^{t}\int_2^4 e^{-\tau}\, d\tau$$

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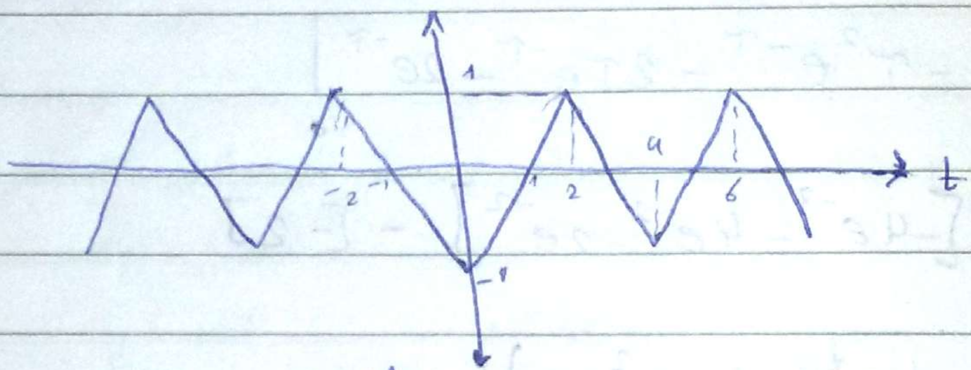
~~Question (b) for the system~~

Find FSTC formial for giver signal:-



$a_0 = 0$ — even + half wave

$b_n = 0$

$$a_n = \frac{8}{T} \int_0^{T/4} x(t) \cos(n w_0 t)\, dt$$

$$= \frac{8}{T} \int_0^{1} (t-1) \cdot \cos(n w_0 t)\, dt$$

= = =

= = =

= = =

n odd value

1 3 5

if find 5th first

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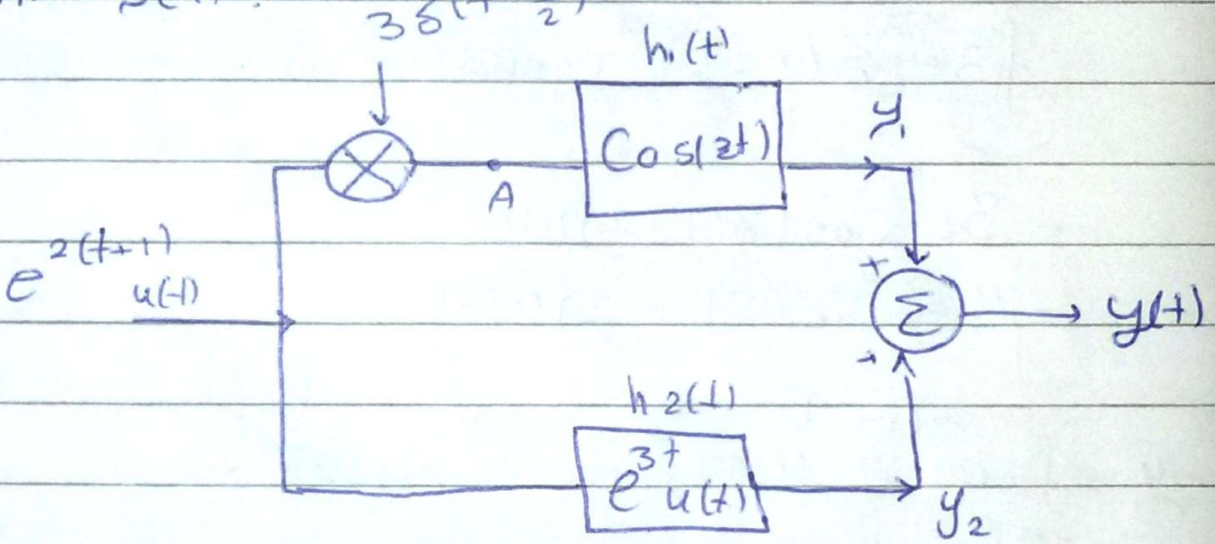
Q.1 _ ⓐ For $h_1(t)$ check linearty ? Prove

② For $h_2(t)$ check stability ? prove

③ find $y(t)$?



Recall / $\int x(t)\,\delta(t-a) = x(a)$

$x(t)\,\delta(t-a) = x(a)\,\delta(t-a)$

① $\cos(3t) \longrightarrow$ not linear #

② $e^{3t}u(t) \longrightarrow t=\beta \Rightarrow e^{3\beta} \neq \infty$ BIBO

③ $y(t) = y_1 + y_2$

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$$y_1 = e^{2(t+1)} u(t) \cdot 3\delta(t-\tfrac{1}{2})$$

$$= 3e^{2(\frac{3}{2})}\delta(t-\tfrac{1}{2}) = 3e^{3}\delta(t-\tfrac{1}{2})$$

$$y_1 = A * h_1$$

$$= \int_{-\infty}^{\infty} 3e^{3}\delta(\tau-\tfrac{1}{2}) * \cos(3(t-\tau))\, d\tau$$

$$= 3e^{3}\cos(3(t-\tfrac{1}{2}))u(t)$$

$$= 3e^{3}\cos(3t-\tfrac{3}{2})u(t)$$

$$y_2 = \int_{-\infty}^{\infty} e^{3(t-\tau)} u(\tau) * e^{2(\tau+1)} u(t-\tau)\, d\tau$$

$$= \int_0^t e^{3t} e^{-3\tau} e^{2\tau} e^{2}\, d\tau$$

$$= e^{2}e^{3t}\int_0^t e^{-\tau}\, d\tau$$

$$= e^{2+3t}\left(-e^{-\tau}\Big|_0^t\right)$$

$$-e^{2+3t}(e^{-t} - e^{0})u(t)$$

$$\quad 1$$

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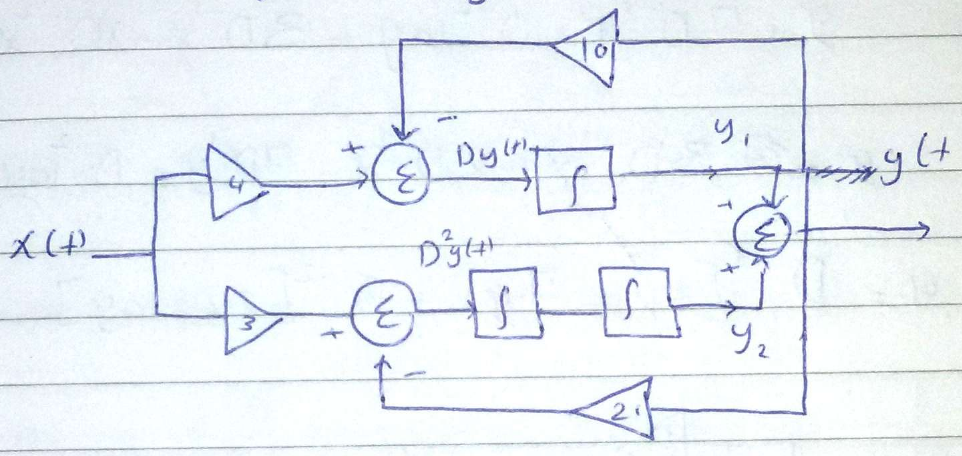
Q2- Find Z-I solution for shown system

Given $y(0) = 0$ / $\dot{y}(0) = 2$



$y(t) = y_1 + y_2$

$y_1 = D^{-1}[4x(t) - 10y(t)]$

$y_2 = D^{-2}[3x(t) - 21y(t)]$

$y = D^{-1}[4x(t) - 10y(t)] + D^{-2}[3x(t) - 21y(t)]$

$D^2y = D4x(t) - D10y(t) + 3x(t) - 21y(t)$

$Dy^2 + D10y(t) + 21y(t) = \underset{0}{3x(t)} + \underset{0}{D4x(t)}$

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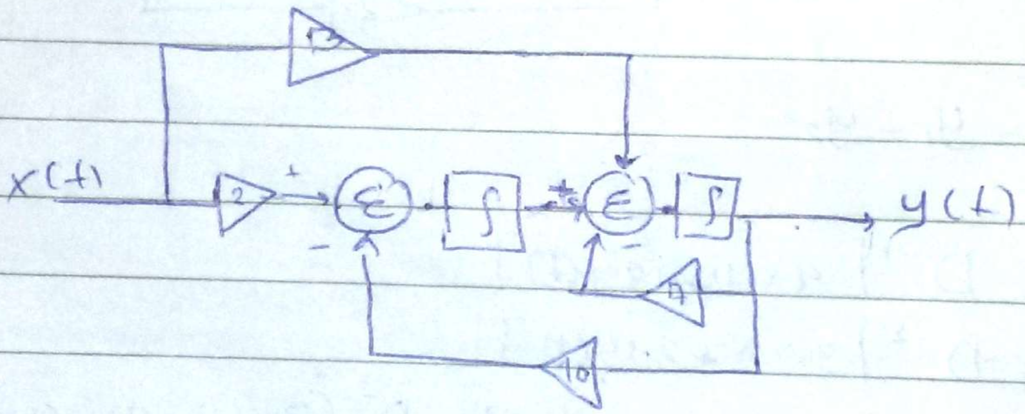
$$Q_2 = D^2y + 7Dy + 10y = 3Dx + 2x$$

$$y + 7D^{-1}y + D^{-2}10y = 3D^{-1}x + 2D^{-2}x$$

$$y = \cancel{\ } 3D^{-1}x + 2D^{-2}x - 7D^{-1}y - D^{-2}10y$$

$$y = D^{-1}[3x - 7y] + D^{-2}[2x - 10y]$$



$$D^{-2}(-10y(t) + 2x(t))$$

$$D^{-1}(-10y(t) + 2x(t)) + (-7y(t) + 3x(t))$$

$$D^{-2}(-10y(t) + 2x(t)) + D^{-1}(-7y(t) + 7x(t))$$

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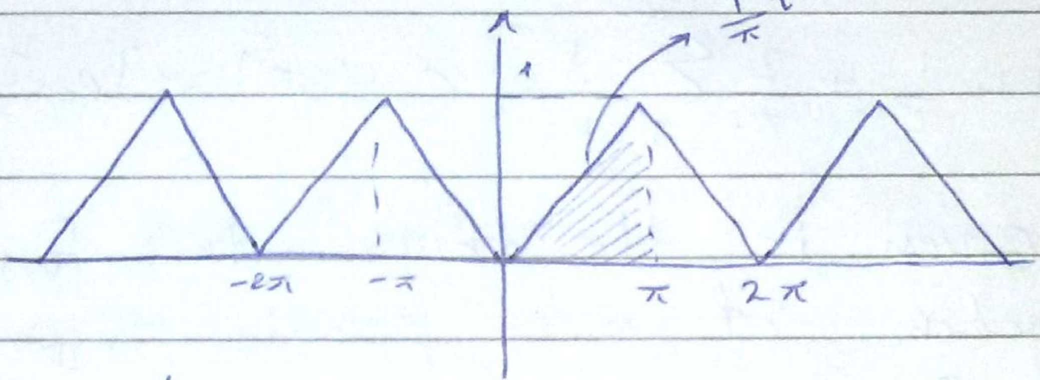
Q find FS for given $x(t)$



even $b_n = 0$ $T_0 = 2\pi$

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)\,dt$$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos(n\omega_0 t)\,dt$$

$$a_0 = \frac{2}{\underset{2\pi}{T_0}}\int_0^{\pi}\frac{1}{\pi}t\,dt = \frac{1}{\pi^2}\int_0^{\pi} t\,dt = \frac{1}{\pi^2}\left(\frac{t^2}{2}\right)_0^{\pi}$$

$$= \frac{1}{\pi^2}\left(\frac{\pi^2}{2}\right) = \boxed{\frac{1}{2}}$$

$$a_n = \frac{\cancel{4}\,2}{\cancel{2}\pi}\int_0^{\pi}\frac{1}{\pi}t\cdot\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{\pi^2}\left[\frac{1}{(n\omega_0)^2}\left[\cos n\omega_0 t + n\omega_0 t\sin(n\omega_0 t)\right]_0^{\pi}\right]$$

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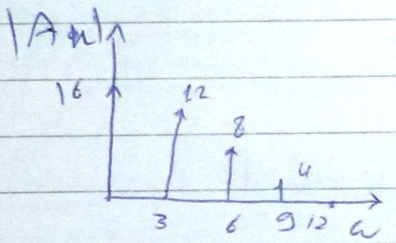
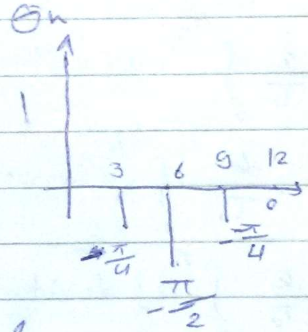
$$a_n = \frac{2}{(\pi n)^2}\left(\cos(n\pi) - 1\right)$$

$$x(t) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{1}^{n} \frac{1}{n^2}\left[\cos(n\pi) - 1\right]\cos n\omega_0 t$$

Q: Given is the trigonomitric Fourier spectrom of some peridec Signal x(t) ?





(a) Sketch the expominal fourier spectrom.

(b) Verify your result anlytically

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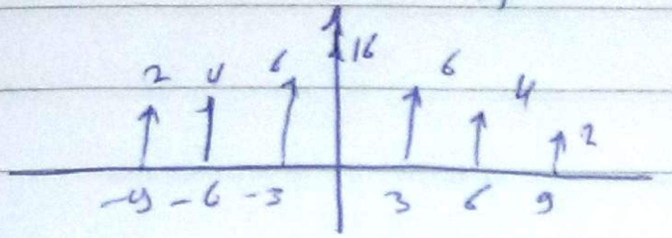
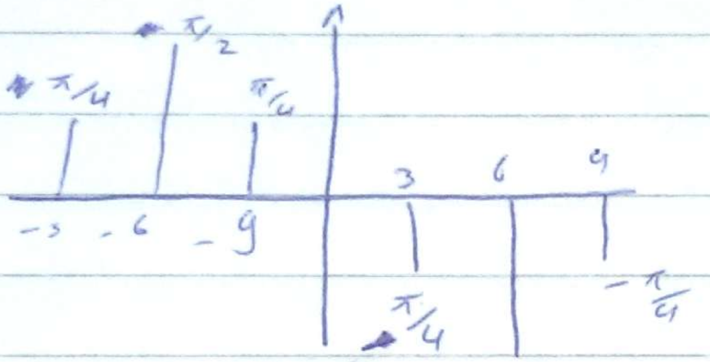


even



$$\underset{\text{DC}}{x(t) = 16} + 6e^{j3t}e^{-j\frac{\pi}{4}} + 6e^{-j3t}e^{j\frac{\pi}{4}} +$$

$$+ 4e^{j6t}e^{-j\frac{\pi}{2}} + 4e^{-j6t}e^{j\frac{\pi}{2}} + 2e^{j9t}e^{-j\pi/4} + 2e^{-j9t}e^{+j\pi/4}$$

$$x(t) = 16 + \underbrace{12\cos(3t - \tfrac{\pi}{4})}_{n=1} + \underbrace{8\cos(6t - \tfrac{\pi}{2})}_{n=2} + \underbrace{4\cos(9t - \tfrac{\pi}{4})}_{n=3}$$

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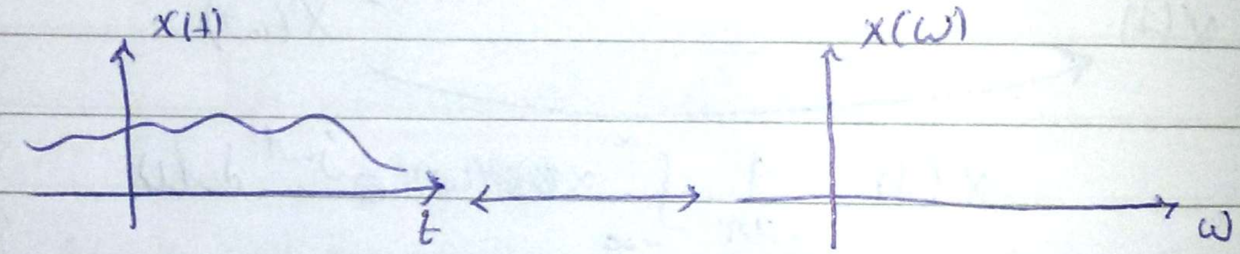
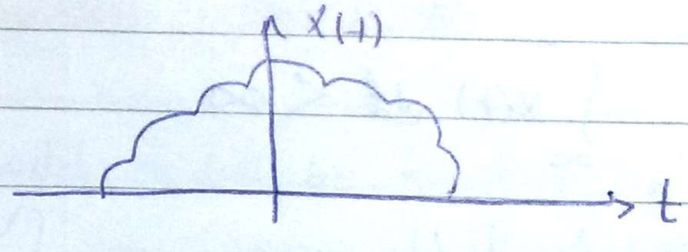
Fourier Transform and linear system /

F.T is a freq. domain transfer that makes Solution: design and analysis of linear System simpley.

F.T is that freq. domain representation of non periodic time domain signals.



$t \longrightarrow \omega$



F.S = F.T → non periodic signal

→ Periodic signal

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$\therefore \; C_n = \frac{1}{T_0}\int_{0}^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$

نفرضوا أنه الـ Signal periodic ونخلي $T \rightarrow \infty$ (تكرر نفسها عدد مالانهاية)

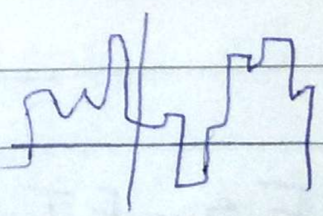
$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) \rightleftarrows X(\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

* Not All signal not periodic can transform

$$\int_{-\infty}^{\infty} x(t)\, dt < \infty$$

and is stable # $\Rightarrow$ 

disconiunte $\neq \infty$

$F\{x(t)\} \longleftrightarrow X(\omega)$

$F^{-1}\{X(\omega)\} \longrightarrow x(t)$

* $X(\omega) = |X(\omega)| e^{\angle X(\omega)}$ مقدار وزاوية

if $X(\omega) = 1 + j\omega$

$$|X(\omega)| = \sqrt{(1)^2 + (\omega)^2}$$

ω is will be positive or nigative it's mean Symmetric (even)

Amlitude => even + positive #

Phase => odd + will be nigative or positive

$$-\angle X(\omega) = \angle X(-\omega)$$

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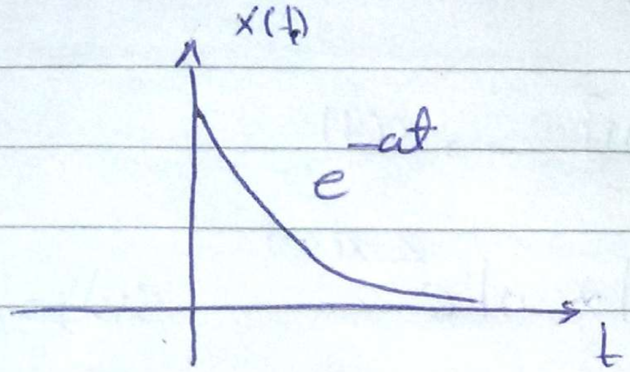
Ex/

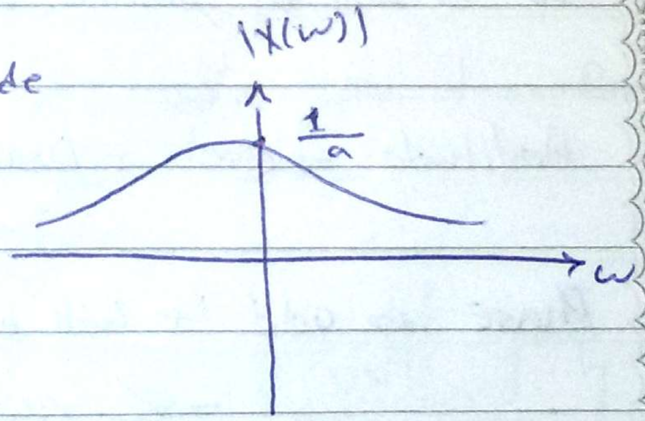
$x(t) = e^{-at}u(t)$

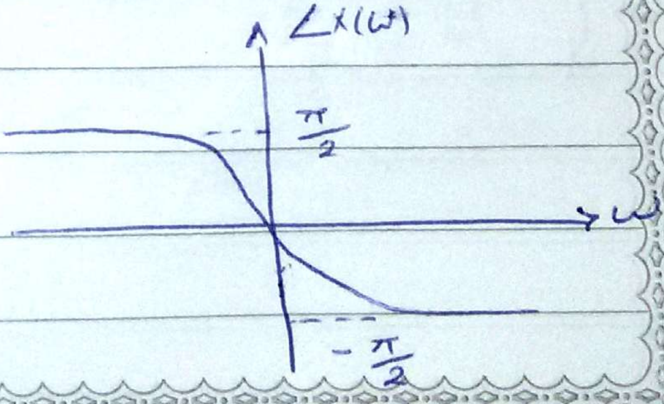


$$\overset{x(t)}{e^{-at}u(t)} = \overset{X(\omega)}{\frac{1}{a+j\omega}} \qquad \text{from table}$$

$$= \left|\frac{1}{\sqrt{a^2+\omega^2}}\right| e^{-\tan^{-1}\left(\frac{\omega}{a}\right)}$$

if $\omega = 0 \Rightarrow \frac{1}{a}$ → magnitude





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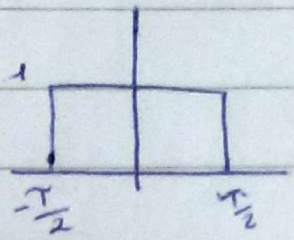
$$x(t) \rightsquigarrow X(\omega)$$

$$e^{-at}u(t) \qquad \frac{1}{a+j\omega}$$

$$\mathrm{rect}\left(\frac{t}{\tau}\right) \qquad \cancel{A}\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$

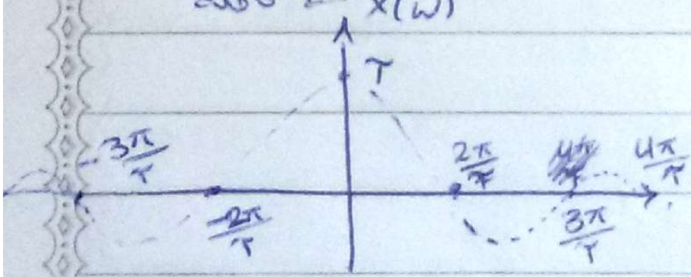


$$\longrightarrow X(\omega) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\,dt = -\frac{1}{j\omega}\left(e^{-j\omega\frac{\tau}{2}} - e^{+j\omega\frac{\tau}{2}}\right)$$

$$= \frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\omega}$$

$$= \frac{\tau\sin\left(\frac{\omega\tau}{2}\right)}{\left(\tau\frac{\omega}{2}\right)} = \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$

كاملة ← $X(\omega)$



if $\omega=0 \Rightarrow \frac{\tau\cancel{2}\sin(0)}{0} = !! \qquad \frac{\tau\sin(0.0\ldots1)}{0.00\ldots1} = 1$

* $\sin(0)=0$, $\sin(0.0000000001) = 0.0000000001$

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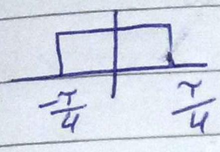
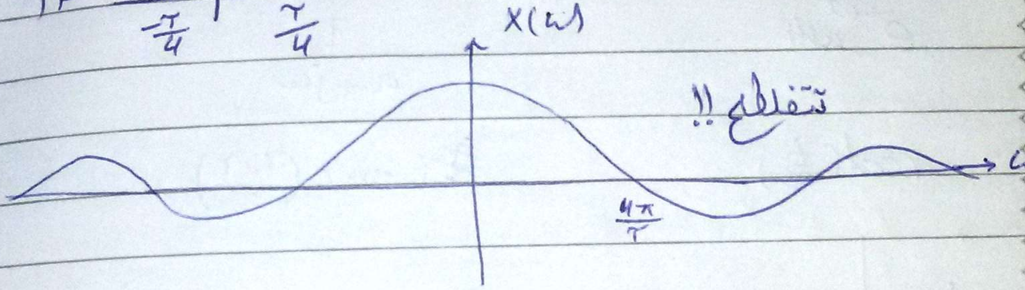
if





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